DEVELOPMENT OF A COLLISION DYNAMICS MODEL FOR SHIP TO SHIP COLLISION IN THE INLAND WATERWAYS OF BANGLADESH

ZOBAIR IBN AWAL

DEPARTMENT OF NAVAL ARCHITECTURE & MARINE ENGINEERING
BANGLADESH UNIVERSITY OF ENGINEERING & TECHNOLOGY (BUET)
DHAKA, BANGLADESH

JUNE 2007

Development of a Collision Dynamics Model for Ship to Ship Collision in the Inland Waterways of Bangladesh

By

Zobair Ibn Awal
(100512001P)

MASTER OF SCIENCE IN NAVAL ARCHITECTURE & MARINE ENGINEERING

Department of Naval Architecture & Marine Engineering
BANGLADESH UNIVERSITY OF ENGINEERING & TECHNOLOGY
Dhaka 1000, BANGLADESH

June 2007

Certificate of Approval

This thesis titled "Development of a Collision Dynamics Model For Ship to Ship Collision in the Inland Waterways of Bangladesh" submitted by Zobair Ibn Awal, Roll No. 100512001P, Session October 2005 to the Department of Naval Architecture & Marine Engineering, Bangladesh University of Engineering & Technology (BUET), Dhaka has been accepted as satisfactory in partial fulfilment of the requirements for the degree of Master of Science in Naval Architecture & Marine Engineering on 11th June 2007.

Board of Examiners

1. ______________________________ **Chairman (Supervisor)**
Dr. M. Rafiqul Islam
Associate Professor
Dept. of Naval Architecture & Marine Engineering
BUET, Dhaka 1000

2. ______________________________ **Member (Ex-officio)**
Dr. M. Sadiqul Baree
Professor & Head
Dept. of Naval Architecture & Marine Engineering
BUET, Dhaka 1000

3. ______________________________ **Member**
Dr. Md. Refayet Ullah
Professor
Dept. of Naval Architecture & Marine Engineering
BUET, Dhaka 1000

4. ______________________________ **Member**
Dr. M. Reaz Hasan Khandoker
Professor
Dept. of Naval Architecture & Marine Engineering
BUET, Dhaka 1000

5. ______________________________ **Member (External)**
Dr. Dipak Kanti Das
Professor
Dept. of Mechanical Engineering
BUET, Dhaka 1000

CANDIDATE'S DECLARATION

It is hereby declared that this thesis or any part of it has not been submitted elsewhere for the award of any degree or diploma.

Zobair

(Candidate)

Zobair Ibn Awal
Roll: 100512001P
Department of Naval Architecture & Marine Engineering
Bangladesh University of Engineering & Technology (BUET)
Dhaka 1000, BANGLADESH

Contents

Acknowledgement

All praises are to the Almighty, who enabled the author to carry out the research and writing up this thesis.

The author feels proud to express his profound appreciation and sincere gratitude to his research supervisor Dr. M. Rafiqul Islam, Associate Professor, Department of Naval Architecture and Marine Engineering, Bangladesh University of Engineering & Technology (BUET) for his keen interest, patience guidance and untiring efforts during every phase of this research works and preparation of this thesis. Without his scholastic supervision and whole-hearted cooperation this thesis would not have been completed by this time.

It is a great opportunity for the author to extend his kind respect and gratefulness to Professor Dr. M. Sadiqul Baree, Head, Department of Naval Architecture and Marine Engineering, Bangladesh University of Engineering and Technology (BUET) for his encouragement and fruitful criticisms in many occasions during the course of study. The author would also like to thank all the teachers of the Department who directly or indirectly assisted and inspired to perform this research work.

A sense of respect and gratitude goes to Professor Dr. Md. Mazharul Hoque, Head of the Department of Civil Engineering and Director of Accident Research Centre (ARC) whose constant support and counsel contributed greatly to the preparation of this thesis. Special thanks are due to the colleagues of ARC for their kind and generous support which the author received during the period of study.

Boundless gratitude is felt and expressed to the parents and relatives without whose blessings and moral support this work would not have been possible.

During the period of study a number of fatal accidents have taken place in Bangladesh to add to the countless loss of lives. The author remembers those tragic moments with great grief and sorrow and would, therefore, like to dedicate the research work to those who lost their valuable lives in marine accidents. This may find a little bit of relief if this research could help preventing such disasters in the future.

The Author

Abstract

This research work attempts to study the accident characteristics in the inland waterways of Bangladesh and to develop a model for ship to ship collision. The collision model is fundamentally divided into two segments namely, model for simulation before collision and model for simulation after collision. In the first part, mathematical formulations are derived for finding the possibility of a collision, determining the location of collision and identification of the contact points on the ships. Using the mathematical relations a Collision Avoidance Chart (CAC) is developed for quick and efficient calculation of determining the collision possibility and thus helping in critical decision making while ships are at collision course and attempting to avoid it. In the later part, a mathematical model is developed to study the kinetic energy losses, collision forces and dynamic responses with respect to different variables such as coefficient of restitution, ship speed, angle of attack, location of hitting, added mass for sway force and others. In the model, expressions for collision forces are derived based on changes in linear momentum. By incorporating the collision force into the equation of motion, which is a linear differential equation with constant coefficients, the dynamic responses are calculated for different collision scenarios. The study considered two different vessels of length 46 meter and 32 meter for conducting the simulations. Results obtained from the mathematical model suggest that collision forces can be reduced significantly by altering the considered variables; e.g. motion amplitudes can be reduced very significantly (as high as eighty five percent) by using materials with lower coefficient of restitution in the fenders and may save ships from capsizing in severe cases. Additional studies also suggest that the risk of capsizing could be eliminated by increasing contact period between the ships. Finally, a number of recommendations have been put forward and further investigations on such models are also proposed.

Chapter 1

INTRODUCTION

1.1 General

The Inland Water Transport (IWT) network is an integral part of the transportation system of Bangladesh and therefore, a safe and efficient water transportation network is absolutely vital for the sustainable development of the country. This chapter describes the key features of the inland water transportation system of Bangladesh with various facts and statistics. The safety situation of this transportation system is also discussed and with that perspective the objective and scope of the present research is presented.

1.2 The Waterways System of Bangladesh

Bangladesh lies at the apex of the Bay of Bengal and has rivers that come down from the surrounding countries and flow through it. Nearly the whole area of the country consists of low and plain lands. According to Banglapedia [1] about 7% surface of the country is covered by a dense 24,000-km long network of inland waterways. Three major river systems and their confluence form the world's largest delta here.

Bangladesh has about 9,000 sq km of territorial waters with a 720-km long coast line and 20,000 sq km of Economic Resources Zone (ERZ) in the sea.

About two-thirds of the land is vulnerable to flooding. Most areas remain under water for two to five months a year. As a result, costs of development and maintenance of roads and railways are high. On the other hand, inland water transport has always been a natural and relatively cheap means of transport in Bangladesh. In certain areas, it is the only mode of transport. Including the country's unclassified routes, the total length of its waterway (700 rivers) is about 13,000 km. Of this, 8,433 km is navigable by larger vessels in the rainy season (5,968 km of which is classified for navigation) while in the dry season about 4,800 km is navigable (classified 3,865 km). Figure 1.1 illustrates the extensive river network of Bangladesh.

Figure 1.1: Extensive water transport network of rivers in Bangladesh.

The IWT system in Bangladesh is both extensive and well connected with the rest of the transport system as well. In terms of traffic intensity, the inland waterways network generates about 1.57 million passenger-kilometres per route-kilometre of waterway [2]. The density of inland ports and terminals is much higher on the inland waterways with approximately 3.7 berthing facilities per 100 route-kilometres. The density of passenger facilities on the inland waterways is also high at around 40 per 100 route-km. According to the Report of the Task Forces on Bangladesh Development Strategies for the 1990’s [3] the inland water transportation system in Bangladesh is the oldest mode of transport that carries nearly one third of the country's total passenger and freight. The report revealed that the private operators own more than 90 percent of the water transports plying in the country. Table 1.1 briefly summarises the inland water transport system of Bangladesh.

Table 1.1: Brief summary of the inland water transport system

No.	Description of the Item	Quantity
1.	Number of registered mechanised vessels	4372
2.	Number of registered non-mechanised vessels	783
3.	Number of country boats: Cargo Boat Passenger Boat	 9800 183000
4.	Number of inland river ports (Developed by BIWTA)	11
5.	Number of coastal inland ports (Developed by BIWTA)	23
6.	Number of ferry ghats (Developed by BIWTA)	7
7.	Number of launch ghats (Including ghats developed by BIWTA)	1563
8.	Number of passenger vessel routes	230

Source: Bangladesh Inland Water Transport Authority (BIWTA)

1.3 Safety of Inland Shipping

With the increase in population and the transportation demand as well the inland waterways of Bangladesh are getting congested day by day. Consequently the numbers of marine accidents are also on the rise. This uprising tendency, however, not only causing serious socio-economic problems in the country but generating concerns in both local and in international community as well. This in turn creates numerous pressures on various aspects of the society and hampers the daily life of common people all across the country.

A number of researches have been conducted on analysing the safety situation of inland shipping system for the last couple of years which are described in the later chapters of this thesis. The studies reveal that the accidents in the inland waterways are very severe in terms of losing valuable lives and resources. The intensity of these accidents, particularly deaths per accident, is higher than accidents in any other modes of transport in Bangladesh. Most of the studies, however, strongly indicate that it is absolutely crucial for all the concerned agencies, individuals and other government or non-governmental organisations to come forward with whatever instruments, measures, and expertise they have and act aggressively to stop these catastrophic incidents.

1.4 Objective and Scope of the Present Research

This research work attempts to reveal the accident characteristics and patterns in the inland waterways of Bangladesh with particular emphasis on the collision type accidents; because these accidents are initiated and executed from man made elements and these accidents are indeed preventable if systematic approaches are being taken. These approaches require involvement of various groups of people including the enforcement agencies, transport owners, transport users, ship designers, ship construction companies and others.

This research is, therefore, the perspective of ship designers for what can be done in preventing the accidents through studying the accident characteristics and comprehend the ships dynamic behaviour in various collision scenarios which will eventually lead to understanding on how to minimize the possibility and the risks of ship collisions.

In this research an attempt has been taken to provide a mathematical formulation in time domain simulation for collision events incorporating several variables so that a realistic study can be conducted in order to understand the causes and mechanisms of collision accidents. This will help the designers designing and operators operating ships which are well prepared to face any adverse situation so that human lives and precious resources will not be at risk.

Chapter 2

CHARACTERISTICS ASSESSMENT OF THE COLLISION ACCIDENTS

2.1 Introduction

In this chapter some literature on maritime safety has been reviewed particularly those which emphasized on the identification of the pattern of accidents. The development of accident database is discussed with the problems and limitations associated with cumulating accident data from various sources. The characteristics of the marine accidents in Bangladesh are studied in detail towards the end of this chapter.

2.2 Literature Review

The transportation safety problem is generally considered a social and behavioural problem rather than a technical problem until lately when a series of catastrophic accidents took place in Bangladesh within a very short span of time. Very recently some researchers emphasized on the technical aspects of ship safety with respect to design and construction of inland vessels. In addition the published research works also emphasized on the identification of the pattern of accidents.

According to “The Report of the Task Forces” [3] one of the fundamental problems relating to the inland shipping safety is that the private operators do not follow the rules and regulations for ship design, construction and operations. This is why their vessels lead to disastrous and fatal accidents in the waterways of the country. Preponderance of private sector in the inland water also makes the assessment of operational efficiency difficult. Also, the private operators do not maintain regular and authentic statistics. A study by Islam [4] on passenger safety was conducted relating the techno-economic evaluation of safe shipping; particularly focusing on the aspects of total cost and ships stability.

An investigation of accidents, damages and cargo losses in inland shipping has been made by Zahanyar and Haque [5] who examined the causes of waterway accidents and made recommendations for the prevention of accidents. Bangladesh Transport Sector Study [6] have classified the waterway accidents focusing on identification of broad types of waterway accidents and suggested several remedial measures commensurate with the classification. BIWTA [7] having constraints of waterway accident investigation system highlighted the safety and stability parameters of the passenger vessels plying within the inland waterways of Bangladesh. In this study technical characteristics of various types of vessels; like year of built, different dimensions, passenger capacity, number of engine, engine type of each registered passenger vessel were analysed by the consultant of BIWTA named Maritime Centre. Detailed discussion and evaluation of the data are yet to be accomplished.

Some research findings by Khalil and Tarafder [8] identified the underlying causes of accidents particularly of the ferry disasters considering 130 accidents. In this study several major reasons of accidents such as inclement weather, overloading, irregular and poor inspection of motor ferries by the marine surveyors, inappropriate weather forecast for inland river ports, lack of well trained masters and crews, undesirable movement of panic-stricken passengers during a crisis, deficiency in adequate number of life saving equipments etc. have been discussed. The study also discussed the issue of design modifications for improving the extra initial stability by upward shift of centre of gravity and thereby preventing the vessel from capsizing in times of emergency.

In a more recent study, Chowdhury [9] developed a GIS based accident information system for water transport accidents and studied the characteristics of the incidents considering 114 accidents. The study, however, recommended future research to be conducted on navigational system integrated with meteorological forecasting systems.

Some statistical analysis has been published very recently by Awal [10] dealing with 197 marine accidents (both passenger and cargo vessels) which showed that majority of the accidents in the inland waterways of Bangladesh occur due to the effect of overloading and/or cyclone (43% of total accidents). It is a fact that the weather condition in Bangladesh is quite unique from other parts of the world; the monsoon season in Bangladesh is quite extensive and many storms (also known as Nor'wester) take place and cause such disasters during this period. The study concluded with a number of recommendations in several categories and also emphasised that the problem remains to be investigated with more insight on particular accident types in order to provide more pragmatic recommendations.

Another investigation by Awal, Islam & Hoque [11] depicted some interesting findings studying accident cases of passenger vessels. It was ominous to note that 56 percent of the passenger vessel accidents in Bangladesh end up in collision due to human error. The second largest cause was discovered to be the loss of stability due to Nor'wester and overloading (21%). The study also showed that 44 percent accidents are being encountered by vessels less than the length of 40 meter and 44 percent in between the length of 40 to 60 meter; therefore, in total 88 percent of the collision accidents occur within the group of vessels of length below 60 meter. Figure 2.1 shows the percentage of accidents according to vessel length.

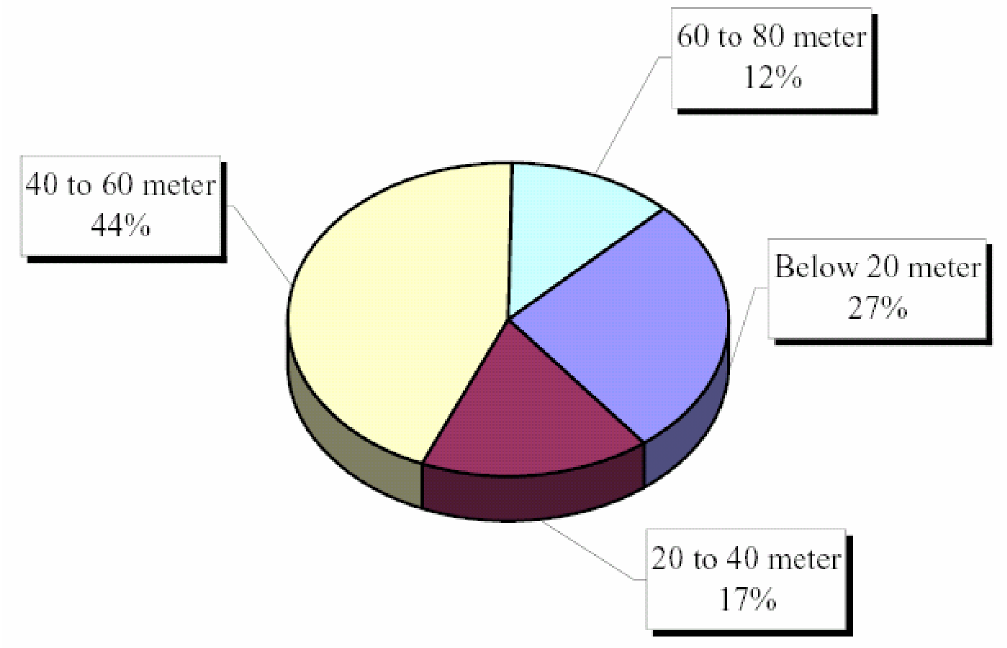


Figure 2.1: Accidents according to length of vessel [11].

The time analysis of these accidents suggests that accidents occur all around the clock but has a slightly higher tendency in between 8 AM to 7 PM, i.e. mostly during the daytime. An analysis using Geographical Information System (GIS) suggested that accidents occur mostly in the surrounding water area of Dhaka and in the river of Meghna. A similar study by Awal [12] showed that vessels having displacements in between 50 tonne to 150 tonne are encountering accidents more than any other group of vessels.

Under the prevailing situation, it becomes quite apparent that collision type accidents are pretty common in Bangladesh and extensive in-depth investigations are required in order to come up with pragmatic solutions to prevent such terrible disasters.

2.3 Data Collection

One of the primary goals of this study is to compile and maintain a technical database, which will work as a tool for in-depth accident analysis and helping researchers to come up with recommendations and solutions in order to prevent such disasters in the future. The accident data were collected from Daily News Papers, reports of Department of Shipping (DOS) and Bangladesh Inland Water Transport Authority (BIWTA). DOS and BIWTA stores accident data essentially for legal purposes and give more emphasis on the parameters related legal issues. Therefore, extractions of scientific data from these reports are very much cumbersome, time consuming and in most of the cases impossible. Therefore, compilation of the accident database takes a pain staking cross matching with different sources of each and individual accidents in order to complete an accident report.

A total of 442 accident cases (occurred in between January 1981 to May 2007) are being considered in this study and a database has been developed in Microsoft Access consisting of 19 different parameters. These parameters are then grouped in to 6 major categories. Figure 2.2 shows the database structure elaborately.

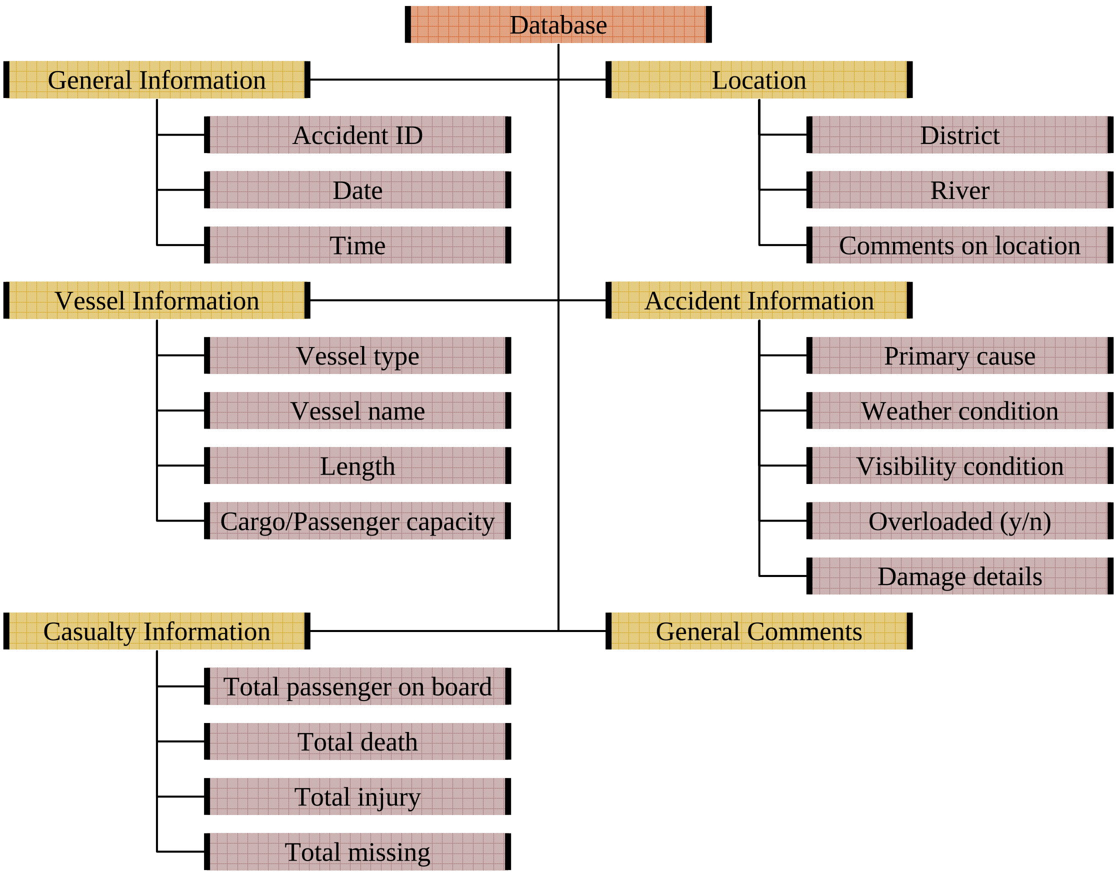


Figure 2.2: Database structure

2.4 Analyses of Accidents

The analyses of the accidents are being performed in Microsoft® Excel worksheets. The results shown in the study are mostly statistical tabulations represented in graphical formats.

2.4.1 Distribution of Accidents on Type Basis

Each year Bangladesh gets affected by Nor'wester. The term Nor'wester is a meteorological term meaning a seasonal storm that appears from the north and western side of the map in the pre-monsoon season. Theses storms appear suddenly with extreme wind force but usually last for a very short duration. They often destroy houses, trees, electric poles, de-stabilise and capsize boats and ships in the rivers. The major cause of accidents in the inland waterways is this Nor'wester as have been seen in Figure 2.3. It is observed from analysis of 442 accident cases that 44 percent of all the accidents in Bangladesh take place due to adverse weather condition coupled with overloading of the vessels.

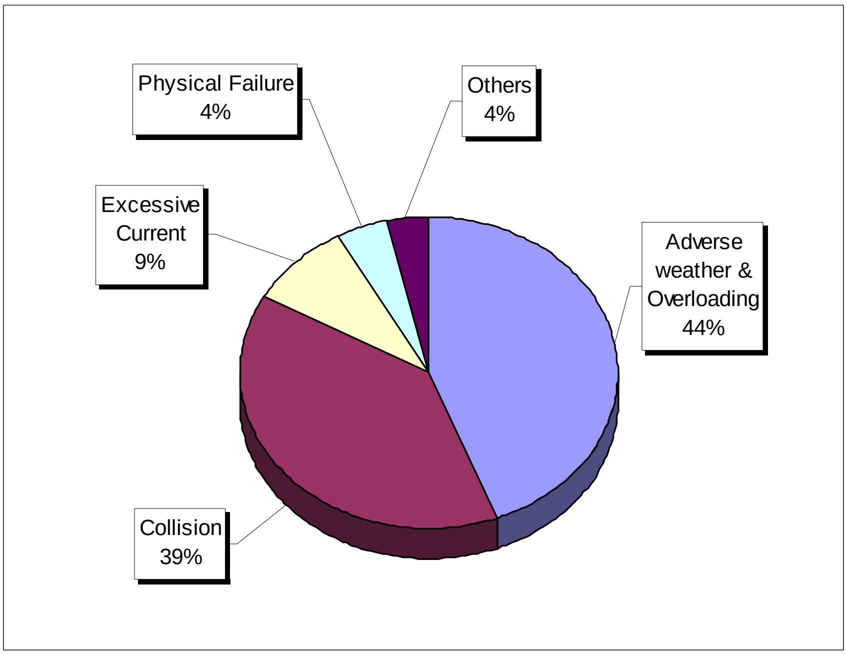


Figure 2.3: Percentage of accident types.

Indeed, the reasons behind the accidents are very much explainable that takes place during the monsoon season. The vessels that face these catastrophic incidents are surely not capable of withstanding the wind and wave forces of the Nor'wester. These accidents are preventable either by not exposing the vessels in the adverse conditions or by constructing and using such novel ships that are literally unsinkable.

Nevertheless, the second major cause is found to be more alarming than any other causes of accidents that have been analysed. It is the collision type accident that is been observed as a serious problem for Bangladesh with a very high rate of 39 percent of all the accidents. It is indeed a very severe percentage and demands a great deal of attention to be looked into the mater.

Bangladesh is generally a highly populated country and the transportation network is basically expected to be congested all around the clock. The growing economy of the country is also increasing the transportation of goods and passengers not only between foreign countries but also within the country as well. It is therefore, explainable why collision type accidents are on the higher side: the more exposed vessels in the limited water areas the more likely collisions will happen.

However, the question might be raised which types of vessels are more affected and what are the particular patterns of these collision accidents. Results suggest that the cargo vessels are involved by the most in comparison to any other type of marine vessels in Bangladesh. The cargo ships are found to be involved with about 80 percent of all the collision accidents and the rest 20 percent of the accidents represents collision between passenger launches and country boats as shown in Figure 2.4.

Theoretically the collision type accidents are of several patterns such as head on collision, side on collision and rear end collision. An attempt has been taken to investigate these accidents but the study has been halted due to limitation of the data. Most of the data sources do not keep records on such technical parameters such as collision angle and point of collision on the ships hull. Therefore, the exact percentages of these patterns of accidents are yet to be known.

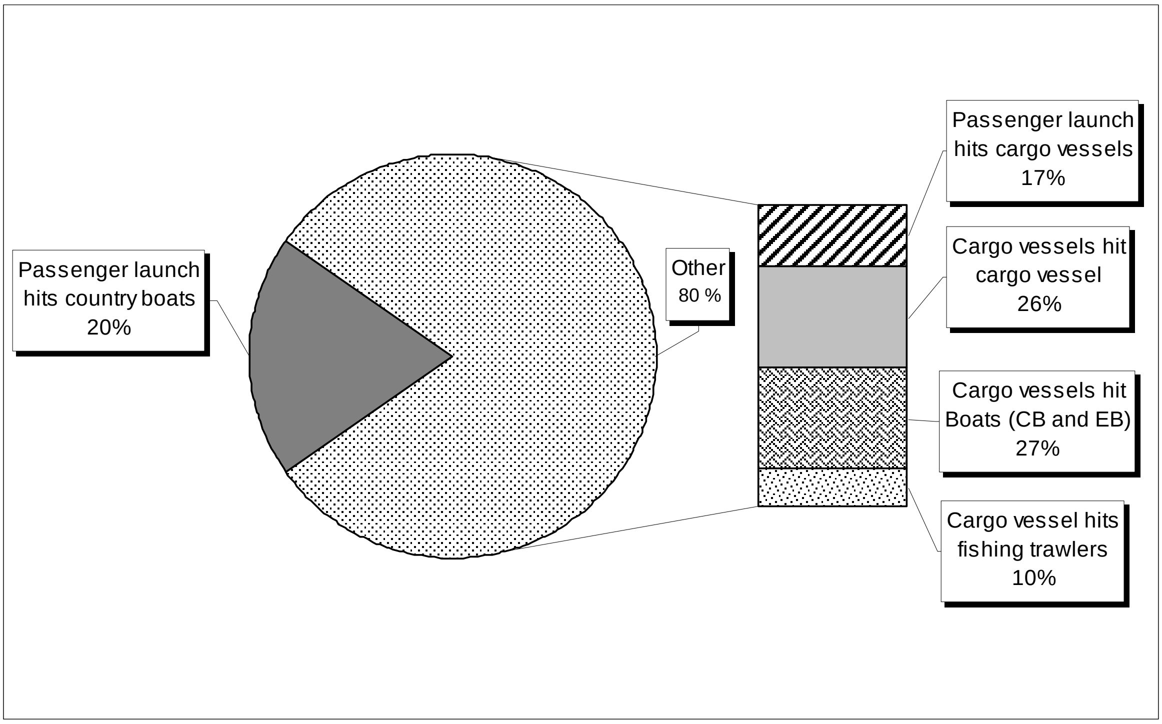


Figure 2.4: Percentage of collision accidents according to vessel types.

2.4.2 Distribution of Accidents on Monthly Basis

The analyses on monthly basis suggest that accidents occur almost all around the years with a bit higher rate during the monsoon season. This is true for accidents between the years 1981 to 2000. The most intriguing fact came up when the monthly analysis of the years between 2001 to 2005 was drawn as it broke all the records of the previous years. This can be seen in Figure 2.5 where it reveals the monthly distribution of the accidents in 5 years segment. The fact that is clearly visible from the figure is that accidents have increased significantly over the last 6 years or so at an alarming rate. It was also clearly visible that from March to August the accident took place more than any other months of the year.

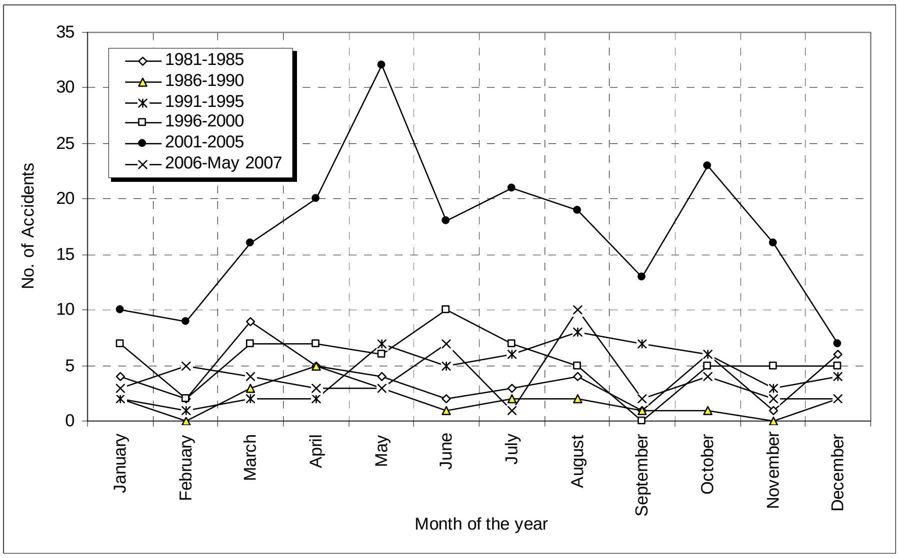


Figure 2.5: Monthly distribution of all accidents represented in 5 years cohort.

In order to analyse the facts further, the collision type accidents are split up from all other accidents and drawn as a separate curve in Figure 2.6. It is observed that during the monsoon season accidents occur above the average line but more interestingly it was observed that collision type accidents occur below the average line during the same period although the variation around the mean line is not as significant (standard deviation 4.74) as for the curve for all types of accidents (standard deviation 11.23).

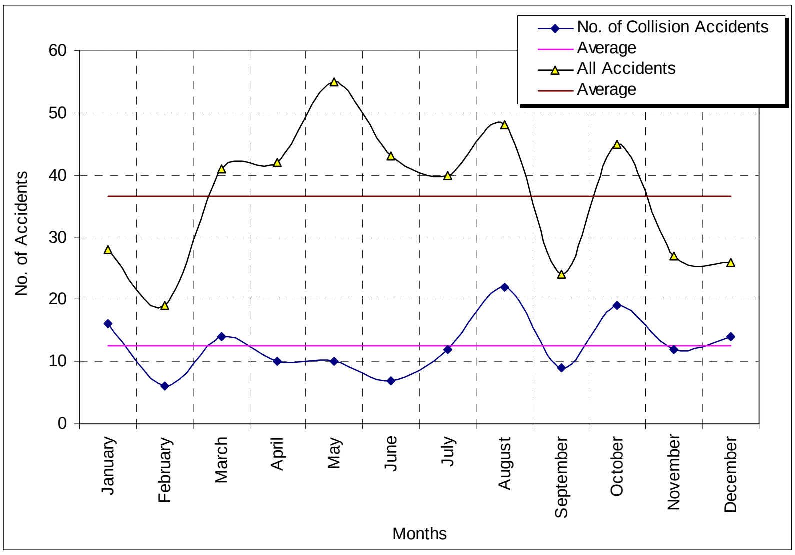


Figure 2.6: All types of accidents and collision type accidents plotted in monthly distribution with respective mean lines.

At this point the question may arise; Is collision type accidents are on the rise as with all other accidents? The answer is obtained through further analysis and is revealed in Figure 2.7; it is seen that there is a drastic increase in collision accidents in the year 2001 to 2005. In addition it is observed that since 2006 already 20 accidents have taken place and if this trend continues at this rate the accident number may reach to 100 at the end of 2010 geometrically.

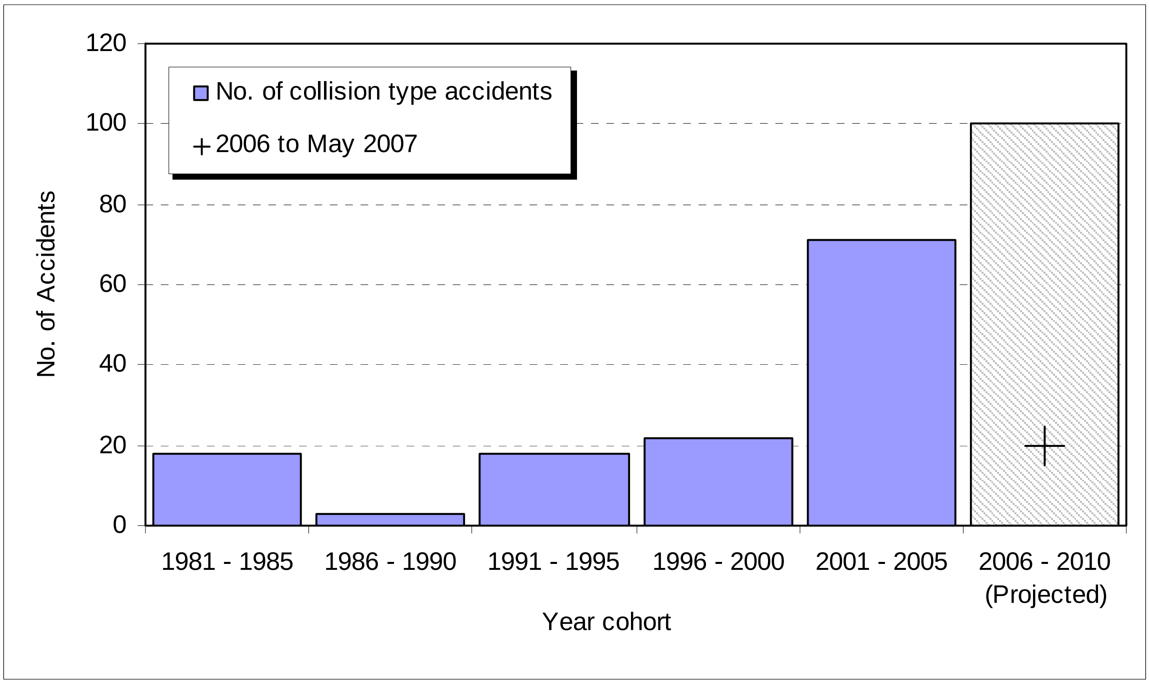


Figure 2.7: Increasing trend of collision type accidents.

2.4.3 Distribution of Accidents on Hourly Basis

So far in the previous analyses it was revealed that collision type accidents generally do not contain any relationship with monsoon season. Logically the next step is to learn that whether the accidents have any relationships with any particular time of the day or not. Analyses on hourly distributions suggest that accidents occur all around the clock except a very small percentage occur at the midnight hours. This is shown in Figure 2.8.

It is observed that among all the six cohorts (4 hours in each cohort) most of the accidents take in place during 8:00 PM to midnight. The reason is very clear because in Bangladesh most the vessels ply without any sophisticated navigational and lighting systems which results limited vision during the trips that they make during night time. Therefore, most of the vessels plying at night are exposed to high risks. However, it is more ominous to notice that the accidents are occurring mostly during the business hours and that is generally in the day time. If the cohorts 4:00 AM to 7:00 PM is summed up it results about 65 percent of all the collision accidents. It appears that although technical deficiencies and limited vision are significant contributors to the accidents but theses elements may not be the only causes behind these catastrophes as accidents are occurring at a high rate during day time.

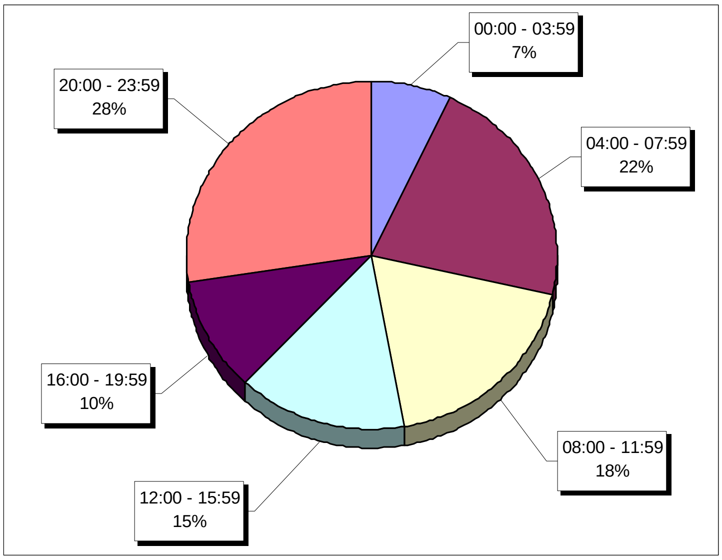
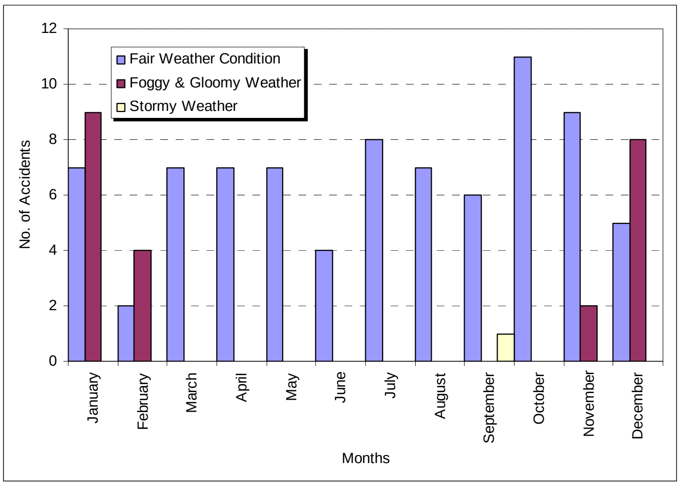


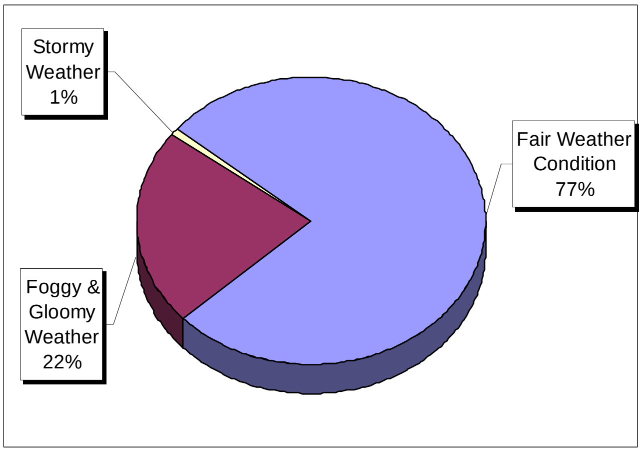
Figure 2.8: Hourly distributions of accidents.

2.4.4 Distribution of Accidents by Weather Condition

It is a generalised fact that collisions take place more during the foggy and gloomy weather conditions, particularly in the winter season in Bangladesh. However, the theory is true only for the months of November, December, January and February as shown in Figure 2.9(a). The analysis suggests that during these months the number of accidents in adverse weather condition is higher than the number of accidents in fair weather condition. However, some interesting findings were brought to light that most of the collisions take place not in foggy or adverse weather conditions but in fair weather condition as shown in Figure 2.9(b). It is clearly seen from the general perspective that most of the accidents (77%) occur in fair weather condition; this eliminates the theory of adverse weather affecting the safety of marine vessels particularly in the cases where fog and mist are assumed to be the only primary cause of collision accidents.



(a)



(b)

Figure 2.9: (a) Accidents according to weather conditions against different months (b) accidents according to weather conditions in percentage.

2.4.5 Distribution of Accidents by Days of the Week

Distribution of accidents by days of the week suggests that accidents tend to occur more during the weekends. As seen from the Figure 2.10 accidents occurred more in Wednesday than any other day of the week. It is a fact that during the weekends people tend to travel for pleasure trips or travel back to their home towns and there is a rush of passengers either by road or by water. This might be a fact that more accidents occur during the weekends as there are more transports exposed into the waters during this particular time. However, the reason for a very high peak at the middle of the week, particularly at the Wednesday is yet to be explained.

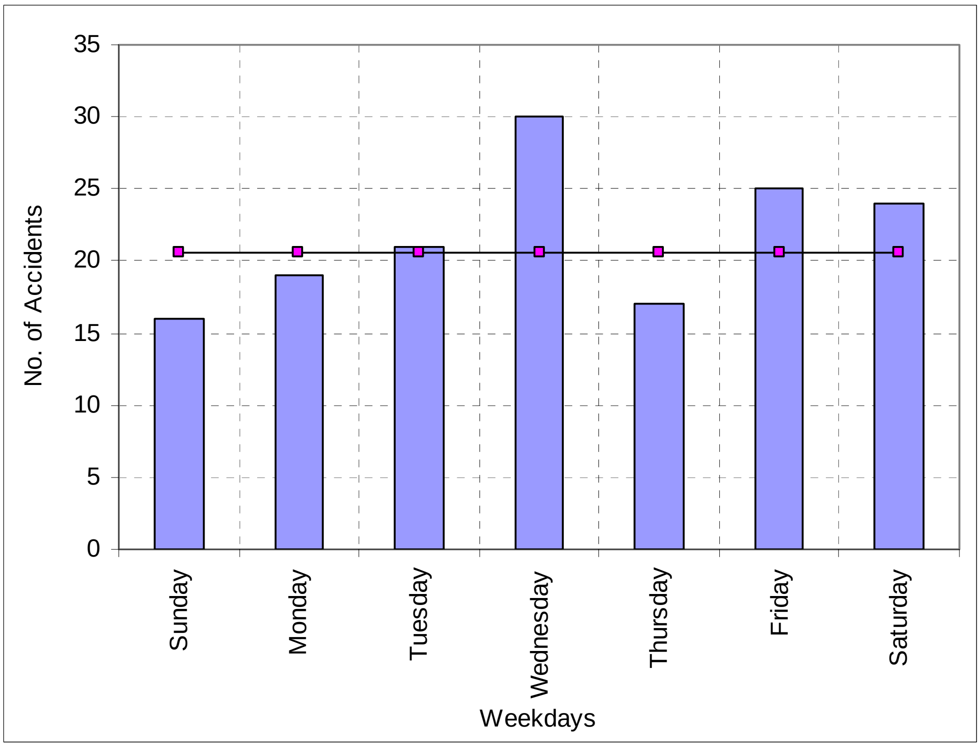


Figure 2.10: Accidents according to days of the week.

2.4.6 District-wise Distribution of Accidents

A very logical question will probably be raised is that where these accidents are happening most? Investigations clearly reveal that accidents occur at very specific districts of the country and at very particular water areas of Bangladesh.

Figure 2.11 shows the district wise accident distribution in Bangladesh. It is observed from the results that the water areas of Narayangangj, Barisal. Chandpur, Munshigangj, Dhaka, Chittagong and Bhola contain the most hazardous water areas of the country in terms of number of collision of marine vessels. Except for Dhaka and Narayangangj the rest of the districts are in the southern part of Bangladesh and possess very large river estuaries.

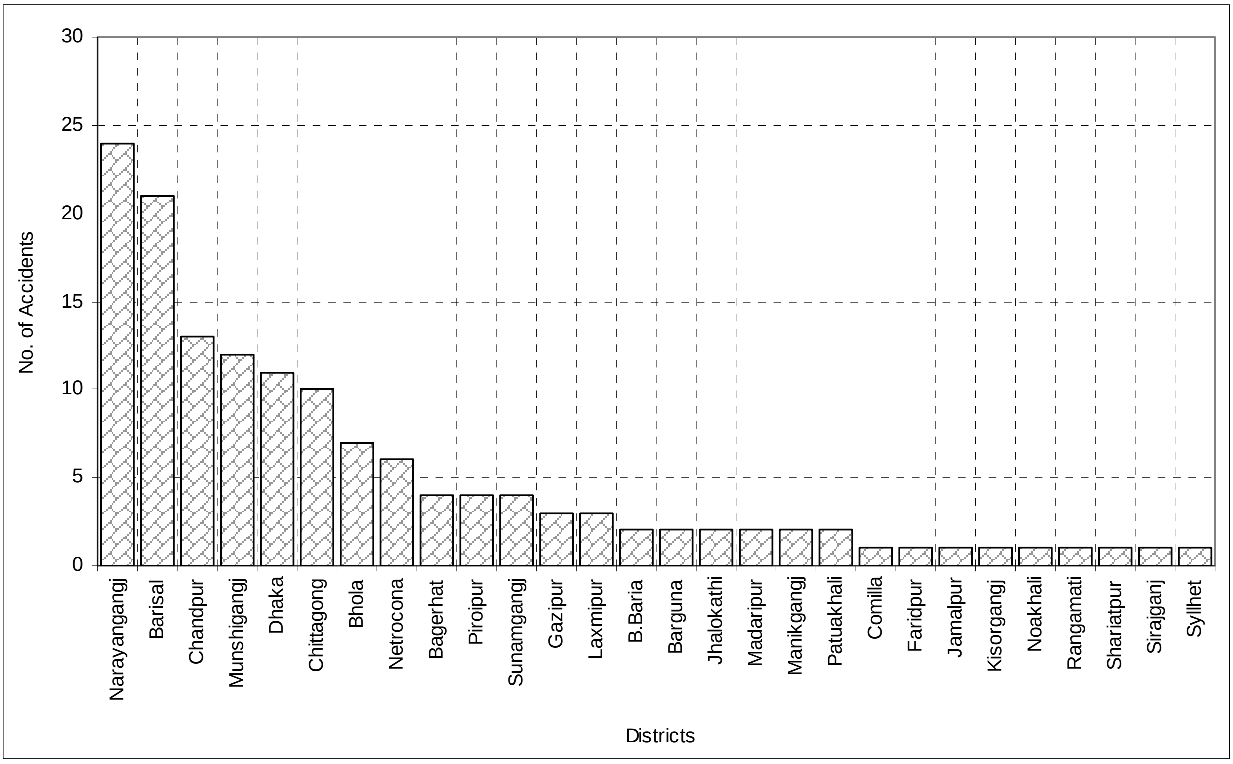


Figure 2.11: District wise distribution of accidents.

Analysis on districts containing ten or more accidents shows that about fifty percent of all these accidents occur in Naryangangj and Barisal districts (Figure 2.12). These two districts of Bangladesh geographically and thus traditionally depend heavily on river networks for business purposes. The primary mode of transportation hence for people residing in these areas are obviously the water crafts. It is therefore, very important that the safety in this particular region remains uncompromised.

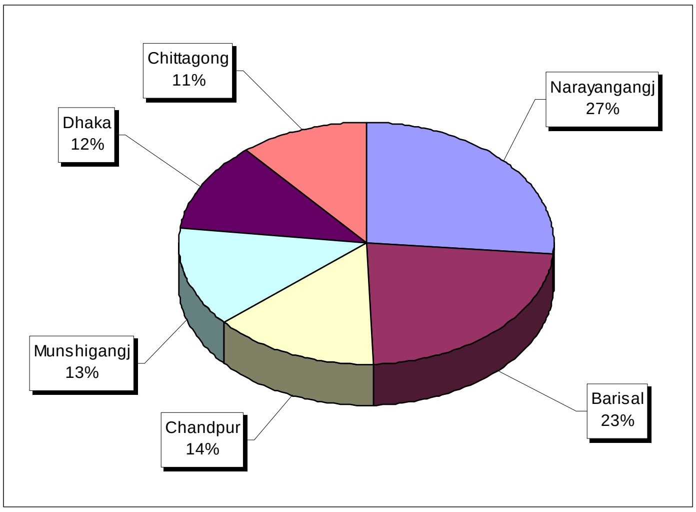


Figure 2.12: Percentage showing district wise accidents.

2.4.7 Total Casualty

Analysis on total casualties suggests that the trend of casualties is on the rise and shows no sign of decreasing at all as it is seen from Figure 2.13. It is interpreted that the number of fatalities and injuries were same during the period 1986 to 2000. However, the number of fatalities has increased significantly in comparison to the number of injuries in the recent years, particularly during 2001 to 2005. It is also notable that although the number of injuries has decreased during this period but the total number of casualties has increased due to a very steep increase in total number of fatalities. Therefore, the accidents are becoming more and more fatal in nature if compared to the accidents of the previous years. Indeed, these findings are very much shocking and require urgent attention from all concerned agencies.

The curve for the number of missing people represents the number of victims whose dead bodies were not recovered. This curve although shows relatively low rate but still adds to the total number of casualties and indeed these are also fatalities.

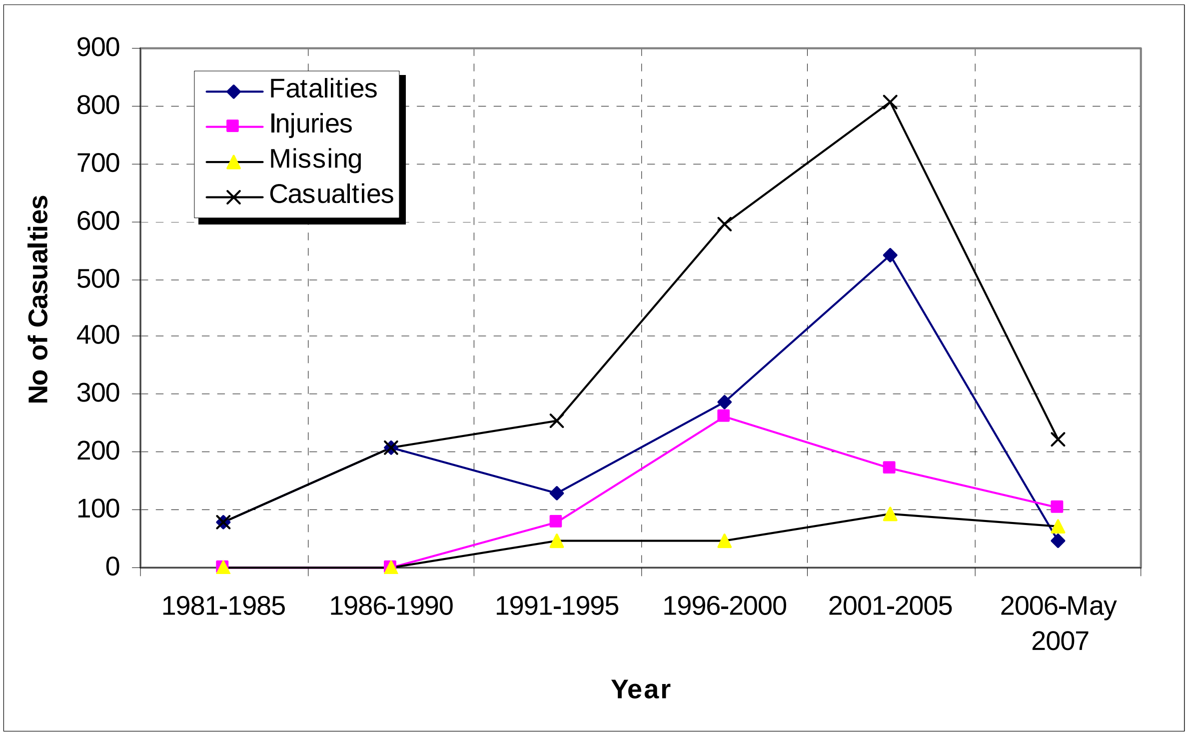


Figure 2.13: Number of casualties, fatalities, injuries and missing since 1981.

2.5 Summary of Analysis

From the analyses so far the following may be summarised:

- The predominant cause of marine accidents in Bangladesh is Nor'wester (44%) and collision between marine vessels (39%).

- Nearly 80 percent of all the collision accidents include the involvement of cargo vessels although they represent a relatively smaller percentage of the total vessel fleet in Bangladesh.

- Accidents are occurring all around the year and almost all around the clock with a bit higher tendency during the business hours. The variation about the mean line suggests that accidents tend to occur more during the weekends and particularly in Wednesdays although accidents occur all around the weekdays.

- 77 percent of accidents occur in fair weather condition and 22 percent occur in foggy & gloomy weather condition, suggesting adverse weather (fog and mist) is not the only cause of collision accidents.

- Around 50 percent of collision takes place in the water areas around Narayangangj and Barisal district. Most of the accidents occur in the southern part of the country.

- Collisions are increasing dramatically over the last 10 years or so without any sign of reducing. Consequently the number of fatalities are increasing significantly and thus the accidents are becoming more and more fatal.

Chapter 3

DEVELOPMENT OF THE MATHEMATICAL MODEL

3.1 Introduction

This chapter discusses the development and formulation of the theoretical ship-ship collision model. The collision model is fundamentally divided into two parts namely, before collision model and after collision model. The chapter starts with a literature review and with reference to the previous research works this concludes with the validation of the developed mathematical model.

3.2 Literature Review

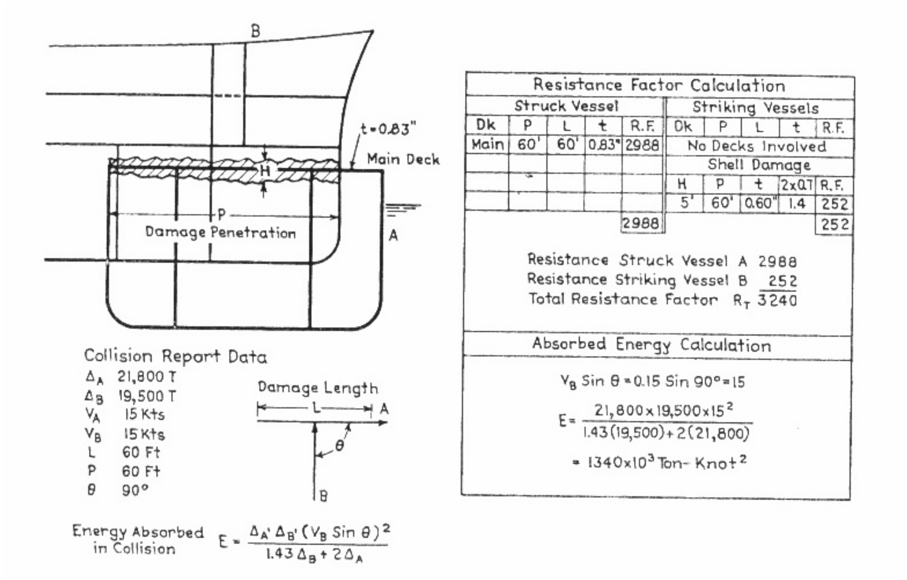
3.2.1 Ship Collision as a Structural Problem

The problem discussed so far in the previous chapters has not only been observed in Bangladesh but also been addressed with due importance all around the world. With the increase in international trade and commerce, accident risks are likely to increase. The risks involved and the consequences associated with a ship-ship collision are extremely high and severe. Particularly the environmental and economical issues create a huge impact in the community when these catastrophic incidents take place.

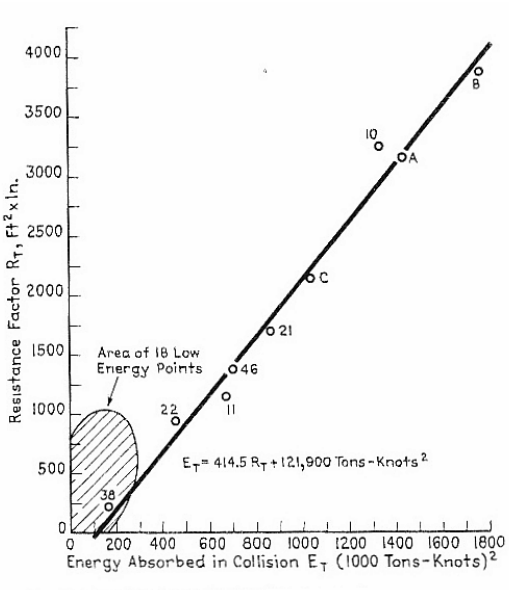
One of the early pioneers to recognise such problems and to conduct mathematical analyses based on empirical models probably was Minorsky [13]. In October 1959, Minorsky published a research paper where he analysed ship collision with reference to protection of nuclear power plant. Minorsky preferred to follow a semi-analytical approach based on the facts of actual collision. The objective of his work was to predict with some degree of accuracy the conditions under which the reactor space remain intact and, consequently, what structural strength should be built into the hull of a nuclear ship outboard of the reactor plant in order to absorb safely a given amount of kinetic energy in a collision.

Minorsky's method consisted of relating the energy dissipated in a collision event to the volume of damaged structure. Actual collisions in which the ship speeds, collision angles, and extent of damage known are used empirically to determine a proportionality constant. This constant relates damage volume to energy dissipation. In the original analysis the collision is assumed to be totally inelastic, and motion is limited to a single degree of freedom. Under these assumptions, a closed form solution for damaged volume obtained by determining the resisting factor and energy absorbed (as shown in Figure 3.1). The research work was later re-validated by many others and is considered one of the best-known empirical collision studies [14].

Zhang [15] in his doctoral research work developed models for ship collisions where collision energy loses, collision forces and structural damages were determined. The analysis procedures were divided into two parts: the external dynamics and the internal mechanics.



(a)



(b)

Figure 3.1: (a) Typical calculation for resistance factor and absorbed energy (b) Empirical correlation between resistance to penetration and energy absorbed in collision [14].

By combining the outer analysis and the inner analysis, a number of examples for full-scale ship collisions were analysed and finally a method relating the absorbed energy and the destroyed material volume was developed and verified. His approach overcome a major drawback of Minorsky's well known method since it takes into account the structural arrangement, the material properties and damage modes. Figure 3.2 shows the revised Minorsky method by Zhang for damage prediction in collision and grounding.

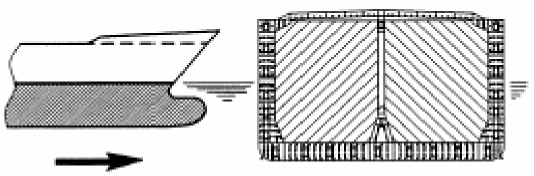
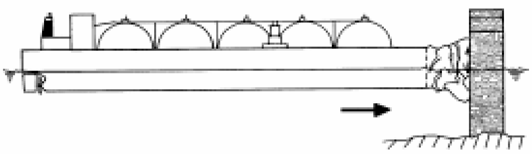
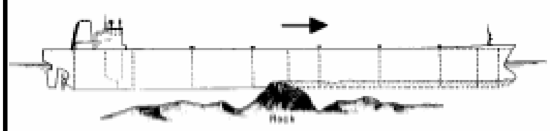
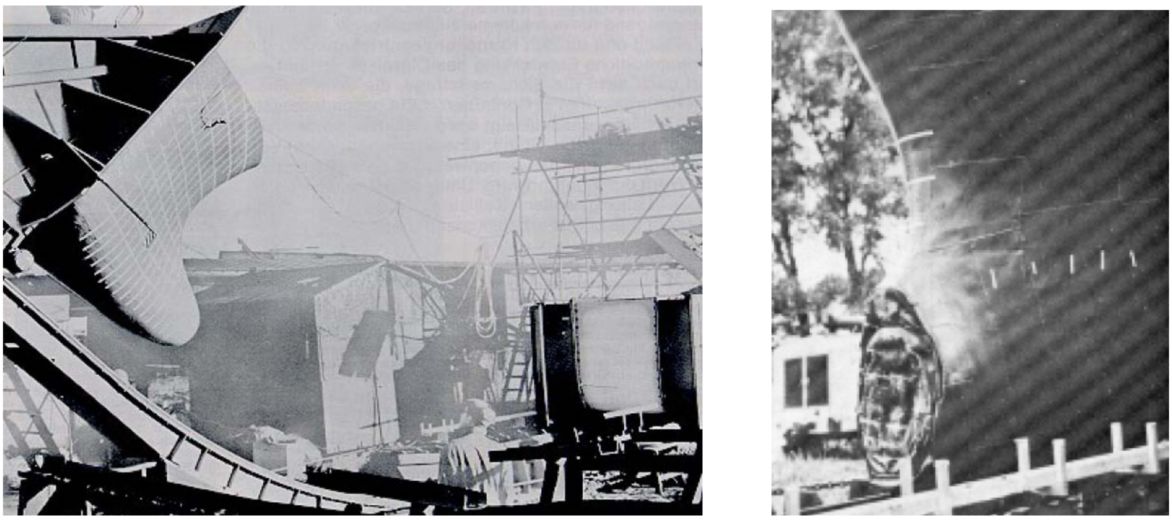
Damage Case	Prediction Method
1. Side collision 	$E_1 = 0.77\sigma_0\varepsilon_c R_{T1} + 3.50(\frac{t}{d})^{0.67}\sigma_0 R_{T2}$
2. Bow crushing 	$E_2 = 3.50(\frac{t}{d})^{0.67}\sigma_0 R_T$
3. Grounding 	$E_3 = 3.21(\frac{t}{l})^{0.6}\sigma_0 R_T$

Figure 3.2: Revised Minorsky method for damage prediction in collision & grounding.

Here in the symbols E_1, E_2 and E_3 represent absorbed energy in the three cases; σ_0 is the flow stress of the material; ε_c is the critical rupture strain of the material; R_{T1}, R_{T2} are the destroyed material volumes; t is the average thickness of the crushed plates; d is average width of the plates in the crushed cross section and l is the critical tearing length.

During the past fifty years a number of model experiments have been carried out in Italy, Germany and Japan. The principal objectives of these tests were to design nuclear powered ships having adequate protection to the nuclear reactor from collision damage. Several authors have given detailed reviews on these experiments, for example, Woisin [16], Amdahl [17], Jones [18], Ellina and Valsgard [19], Samuaelides [20] and Pedersen et al. [21].

During the period of 1967 to 1976, 12 model ship collision tests were carried out in Germany (Woisin, 1979). The model scales range from 1/12 to 1/7.5. The test setup illustrates the striking bow running down from an inclined railway path hitting the side shell of a ship as shown in Figure 3.3 (a) along with a typical damaged bow in Figure 3.3 (b).



(a) (b)

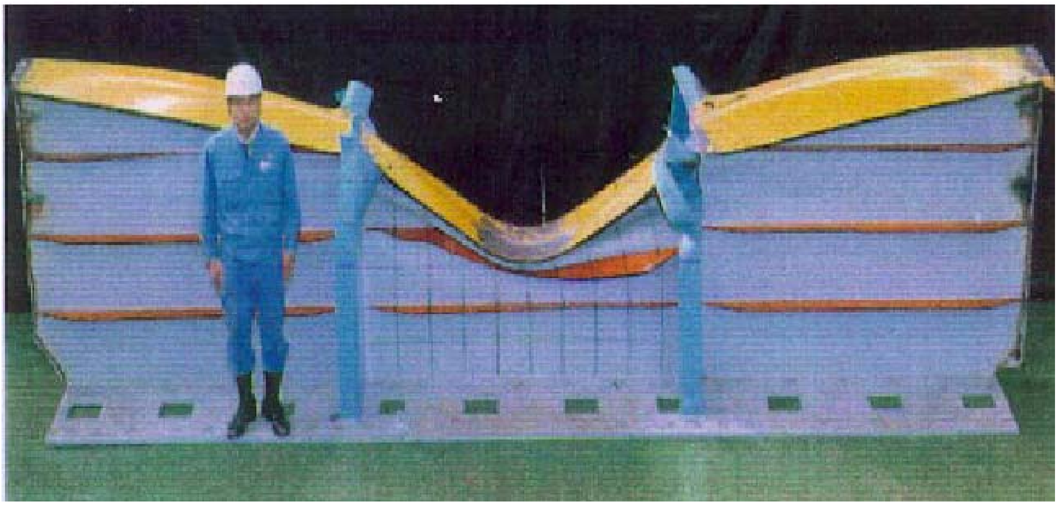
Figure 3.3: (a) Experimental setup described by Woisin [16] and (b) damaged bow after collision testing

During 1991-1997 several research projects were taken on the prediction methodology of tanker structural failure focusing two main aspects: one is the dynamic process of structural damage caused by collisions or grounding, and the other was the resulting process of structural oil spill and/or water ingress through damaged hull. A series of full-scale ship collision experiments was carried out in the Netherlands in co-operation with Japan. A full-scale collision experiment conducted in 1998 in the Netherlands can be viewed in Figure 3.4.

Figure 3.4: Full scale collision experiment in the Netherlands.

A series of similar large-scale dynamic side collision experiments was also conducted in Japan and Denmark during 1992 to 1996. A full-scale dynamic collision test of a 40,000 dead weight tanker was carried out by Qvist et al. [22]. A 2.75 ton rigid bass was used to simulate a striking bow, which was dropped from a height of 5 meters simulating a striking velocity of 20 knots. One of the tested models after collision is shown in Figure 3.5 (a). A similar experiment was carried out in Japan on the models of large oil tankers for simulating side collision. Figure 3.5 (b) reveals one of the test experiments.

(a)



(b)

Figure 3.5: (a) Full-scale side model after collision experiment in Denmark and (b) Dynamic side collision experiment in Japan [22].

Interestingly all these experimental tests were primarily intended to investigate the structural performance with respect to providing watertight integrity, safeguarding the extremely valuable passenger, cargo, and other important goods. So far none of the research works have emphasised the importance of the dynamic characteristics of the ships during a collision event.

In some recent investigations, powerful computers are used to model collision scenarios using finite elements. Simulation programs started to run into the computers and events could be seen in time frame, second by second. According to Dimitris et al. [23] there are two major questions that naval engineers working on ship collisions should approach: One concerns the simulation of ship collisions and the prediction of the damages, which occur during the incident. The other is the identification of collision scenario or scenarios, which the ship under consideration should be checked against in order to assess her capacity to withstand collision loads. Dimitris in his work used extensive finite element codes for collision simulation. Figure 3.6 shows one of the simulation scenarios.

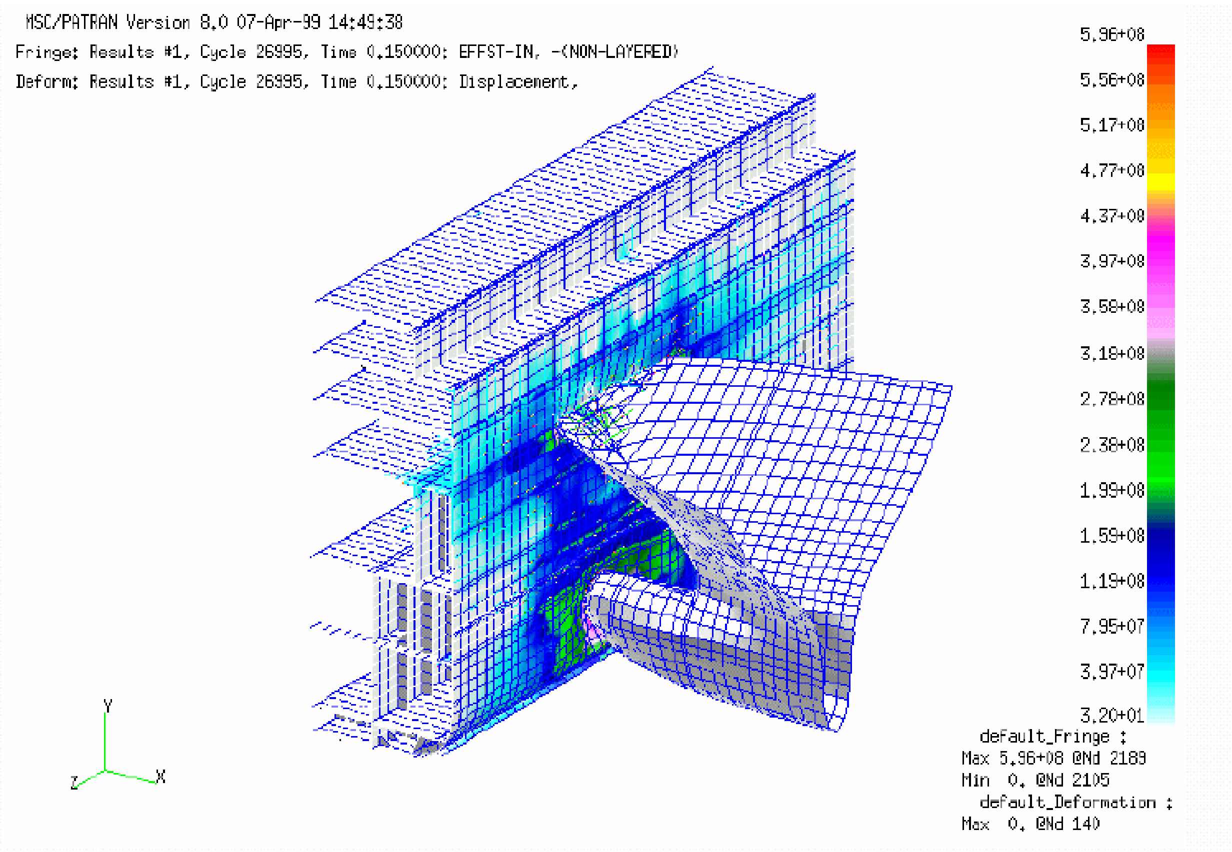


Figure 3.6: Relative position of the struck area and striking bow at 0.15secs: Equivalent stress [23].

Brown and Chen [24], Brown et.al [25] and Chen [26] conducted extensive work in developing Simplified Collision Model (SIMCOL) based on solutions of external dynamics and internal deformation mechanics in time domain simulations for the rapid prediction of collision damage in probabilistic analysis. The external sub-model used a three-degree of freedom system for ship dynamics. The internal sub-model determined reacting forces from side and bulkhead structures using mechanisms adapted from Rosenblatt and McDermott [27,28], and absorbed energy by decks, bottoms and stringers calculated using the Minorsky's correlation as modified by Reardon and Sprung [29].

A computer program DAMAGE was developed at the Massachusetts Institute of Technology (MIT) under the Joint MIT-Industry Program on Tanker Safety. The program DAMAGE Version 5.0 can be used to predict structural damage in the accident scenarios such as ship grounding on a conical rock (rigid rock, deferrable bottom), ship-ship collision (deferrable side, deferrable bow). A major advantage of DAMAGE is that the theoretical models are hidden behind a modern graphical user interface (GUI) [30].

3.2.2 Research on Ship Dynamics

Over the years there have been considerable developments in the ship dynamics problems considering wave and wind loads in two dimensional strip theory method viz. Salvesen et. al [31] and Vugts [32]. A notable work in such areas is also done by Bhattacharyya [33] based on strip theory and his works are considered remarkable for computing wave loads on ships even these days. Advancement in computer technology have made possible the development of new classes of three-dimensional numerical tools for analysing problems which were almost impossible to conduct in just a decade ago. Some researches have been done on floating bodies by 3-D source distribution method without forward speed [34-36]. Other research works on 3-D source distribution method with forward speed have been done lately using large panels by Inglis and Price [37, 38]. Recently Islam et. al [39] developed methods for calculating the hydrodynamic coefficients using panel method based on potential theory which are capable of dealing with a large number of panels of very small size. Although there are sufficient developments in computing the dynamic responses due to wave and wind loads but very limited research works have been conduced in determining other loads such as collision, grounding and others.

3.3 Development of Theoretical Model

The research works that have been done so far over the years mostly concentrated on determining the structural responses during collision events and there have been considerable advances in developing methodologies and formulations of determining the collision damages. However, most of these research works ignored the dynamic responses of ships as a whole and the aspects of ship stability with reference to capsizing due to excessive rolling. The present study is, therefore, dedicated to formulate and investigate as well the ships dynamic characteristics over time i.e time domain motion simulation of ships.

The mathematical model developed in this study can be divided into several segments with reference to the time domain analysis. Such as (1) Before Collision Model and (2) During and After collision Model. However, for both of the segments there are some fundamental assumptions adopted and these are as follows:

1. It is assumed, only while determining the hitting position at the struck ship abaft the bow, that the ships are straight line objects and their breadths and curved body shapes are ignored.

2. There is no friction or sliding between the striking and struck ship. The ships get separated from each other after the collision.

3. It is also assumed that the ships do not encounter a second collision after being hit by each other at the first instance so that the ships can have free motions in space after the collision.

4. For time domain simulation the collision time (contact period between the two ships) is assumed as one second. However, for additional analyses the contact period has been considered as a variable in between 1 to 5 second.

5. It is also assumed while dealing with the equation of motion that there are no wave or wind forces before and after collision.

3.3.1 Model for Simulation Before Collision

It is considered that two ships are plying in waters each having their particular forward speed and heading. Knowing these parameters there arise four questions that are considered fundamental for the mathematical formulations:

1. With the given speeds and headings, will there be a collision between two ships?

2. If there is a collision, where will it take place (i.e the location)?

3. Which ship hits the other?

4. Which part of the ship along its length the hitting takes place?

In order to find the answers of these questions it is crucial to locate the point of intersection of the paths of the vessels. If it is assumed that the given vessels keep their course unaltered, there will be a certain point at which their straight-line paths of will intersect each other. This point may be defined as the Point of Intersection of the Paths. This is, however, not necessarily be the point at which the ships get struck rather it gives an idea about the location on the water certain distance away from the ships where the accident will take place.

Since it is assumed that the ships remain in their heading and do not alter their course, their paths can therefore, be expressed as straight lines. Considering Figure 3.7 let the paths of the vessels be:

$$y = m_A x + c_A \qquad \text{(For Ship A)}$$

$$y = m_B x + c_B \qquad \text{(For Ship B)}$$

Where,

m_A & c_A and m_B & c_B are the slopes and constants of Ship A and B respectively which depend on the ships heading and relative position on the co-ordinate system.

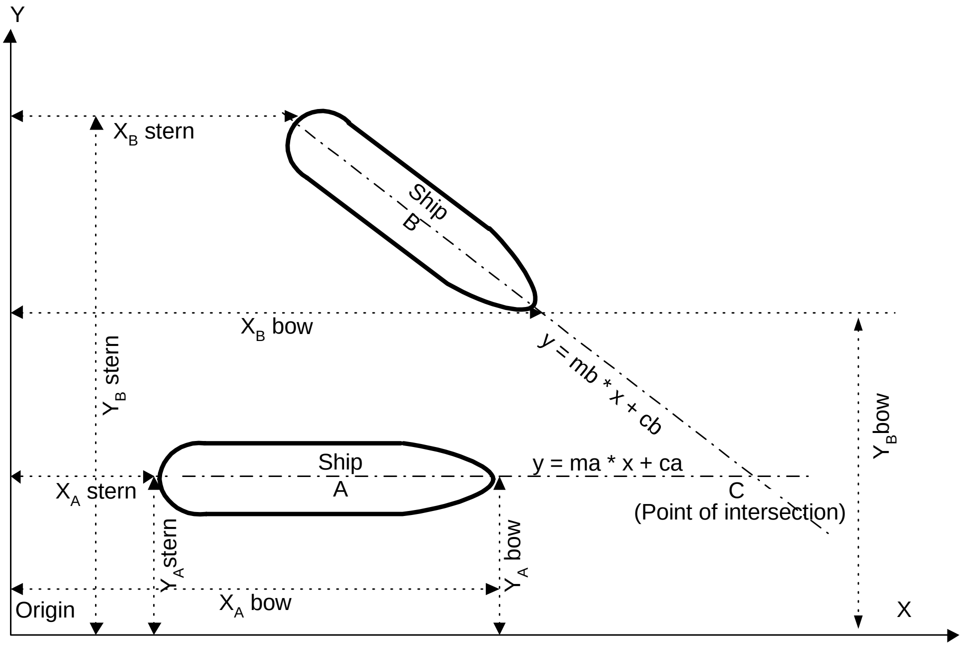


Figure 3.7: Co-ordinate system for simulation before collision.

The slopes of these straight lines are obtained from the relative position of their sterns and bows in Cartesian co-ordinates.

$$m_A = \frac{y_{Abow} - y_{Astern}}{x_{Abow} - x_{Astern}} \qquad \text{(For Ship A)}$$

$$m_B = \frac{y_{Bbow} - y_{Bstern}}{x_{Bbow} - x_{Bstern}} \qquad \text{(For Ship B)}$$

Where,

x_{Abow} = X co-ordinate of bow of Ship A

y_{Abow} = Y co-ordinate of bow of Ship A

x_{Bbow} = X co-ordinate of bow of Ship B

x_{Bbow} = Y co-ordinate of bow of Ship B

x_{Astern} = X co-ordinate of stern of Ship A

y_{Astern} = Y co-ordinate of stern of Ship A

x_{Bstern} = X co-ordinate of stern of Ship B

x_{Bstern} = Y co-ordinate of stern of Ship B

Putting these into the straight-line equation the constants are obtained as:

$$c_A = y_{Abow} - m_A x_{Abow} \qquad \text{(For Ship A)}$$

$$c_B = y_{Bbow} - m_B x_{Bbow} \qquad \text{(For Ship B)}$$

Let the point of intersection be defined by C (X_c, Y_c) and it is obtained as,

$$X_C = -\left(\frac{c_A - c_B}{m_A - m_B}\right)$$

$$Y_C = \frac{m_B c_A - m_A c_B}{m_A - m_B}$$

Knowing the point of intersection gives the probable idea of the location of collision. However, the incident of collision can still be avoided by altering the speeds of the vessels and not altering their courses. Therefore, it is important to know whether both the ships occupy the point of intersection at the same time or not. If they occupy the point at the same time, the collision is inevitable. On the other hand, if any of the ship passes through before arrival of the other or arrives late while allowing the other to pass through, the collision could be avoided. This critical situation may well be similar to when vessels lose control of their rudder on a collision course and have only the freedom of varying their respective speeds.

The problem now requires the period for which the ships occupy the point of intersection to be known. Let the measured parameters of relative positions of the ships with respect to origin were taken at time t = 0. If one of the ship's bow (say Ship A) arrives at the intersection point at time $TA_{arrival}$ and the stern leaves the point at time $TA_{departure}$, then the occupation duration is the difference between $TA_{departure}$ and $TA_{arrival}$. In such a case if a collision is to take place, the other ship must arrive the point in between time $TA_{departure}$ and $TA_{arrival}$. Therefore, it is essential to know the arrival and departure time of both of the ships at the point of intersection or "Collision Zone". An example is shown in

Figure 3.8 where the scenario is depicted. In the first case both of the ships occupy the collision zone at the same time as seen by the overlapping time line of the ships. However, in the second case the ships occupy the collision zone at different time and avoid the collision.

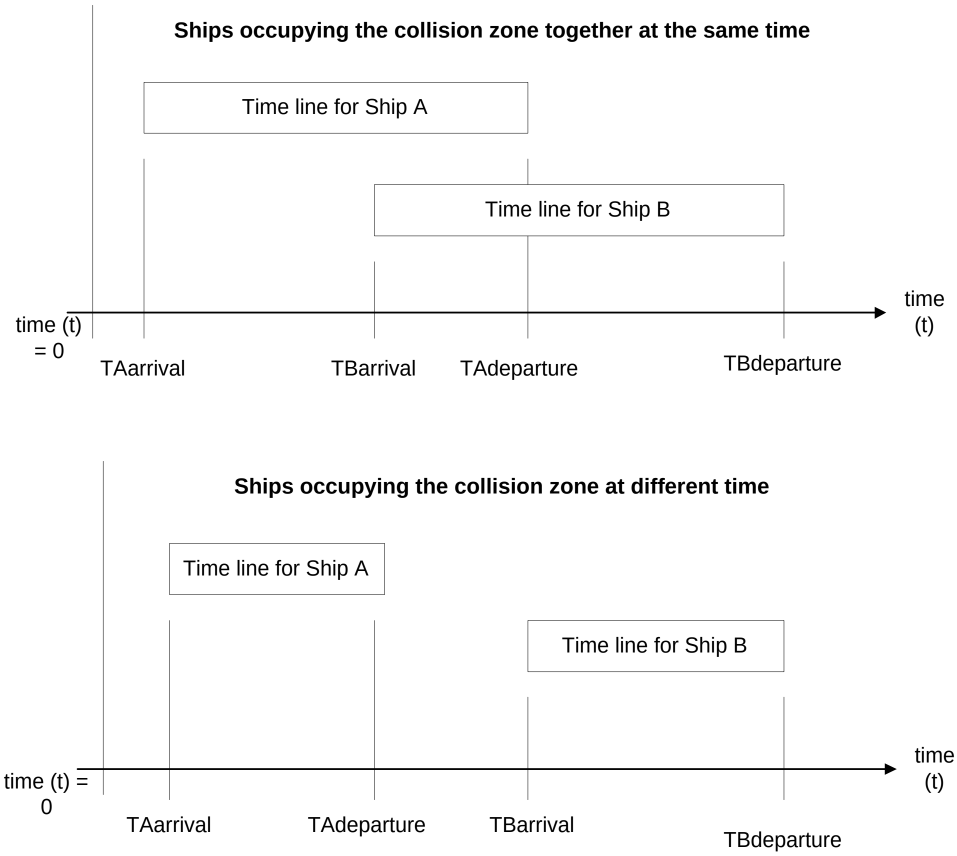


Figure 3.8: Time line description of the ships occupying the collision zone.

To determine the arrival time and departure time of the ships ($TA_{arrival}$, $TB_{arrival}$, $TA_{departure}$, $TB_{departure}$) the following calculations are performed for ship A,

Speed in the X-direction x Arrival Time = Horizontal distance between bow of the ship & Xc

i.e $$V_A \cos m_A \times TA_{arrival} = X_c - x_{Abow}$$

or, $$TA_{arrival} = \frac{X_c - x_{Abow}}{V_A \cos m_A}$$

Again speed in the X-direction x Departure Time = Horizontal distance between stern of the ship & Xc

i.e $$V_A \cos m_A \times TA_{departure} = X_c - x_{Astern}$$

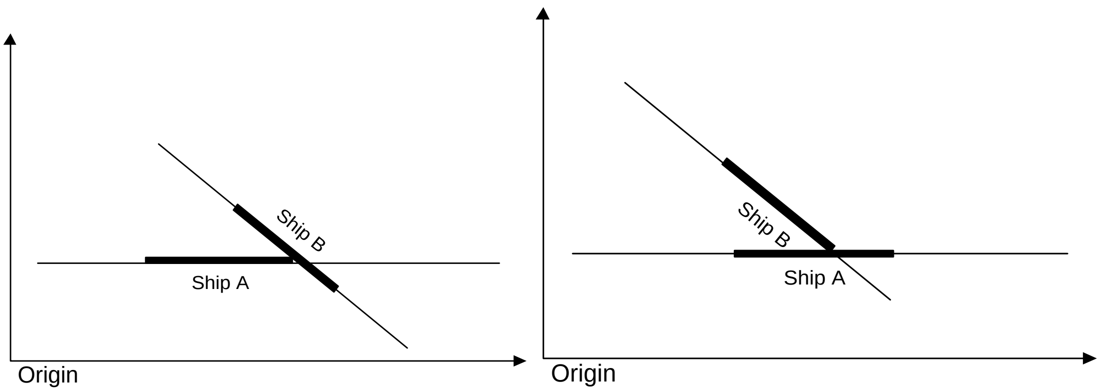
or $$TA_{departure} = \frac{X_c - x_{Astern}}{V_A \cos m_A}$$

Similarly for ship B the following are obtained:

$$TB_{arrival} = \frac{X_c - x_{Bbow}}{V_B \cos m_B}$$

$$TB_{departure} = \frac{X_c - x_{Bstern}}{V_B \cos m_B}$$

If Ship A arrives at the point of intersection before Ship B and Ship A doesn't depart before arrival of Ship B (i.e. $TA_{arrival} < TB_{arrival} < TA_{departure}$), in such a case Ship B hits Ship A and the collision occurs at $TB_{arrival}$. On the other hand, if Ship B arrives at the point of intersection before Ship A and Ship B doesn't depart before arrival of Ship A (i.e. $TB_{arrival} < TA_{arrival} < TB_{departure}$), in such a case Ship A hits Ship B and the collision occurs at $TA_{arrival}$. Figure 3.9 depicts the two situations.



(a) (b)

Figure 3.9: (a) Ship A hits Ship B and (b) Ship B hits Ship A.

Let the time be T_{hit} when the collision takes place. When Ship A Hits Ship B, the position of the point of hitting at Ship B abaft bow is given by,

$$H = \sqrt{\{X_C - (x_{Bbow} + V_B \cos\phi \times T_{hit})\}^2 + \{Y_C - (y_{Bbow} + V_B \sin\phi \times T_{hit})\}^2}$$

On the other hand when Ship B Hits Ship A, the position of the point of hitting at Ship A abaft bow is given by,

$$H = (x_{Abow} + V_A \times T_{hit}) - X_C$$

Using the mentioned mathematical formulations a computer program has been developed in Microsoft (TM) Excel as shown in Figure 3.10.

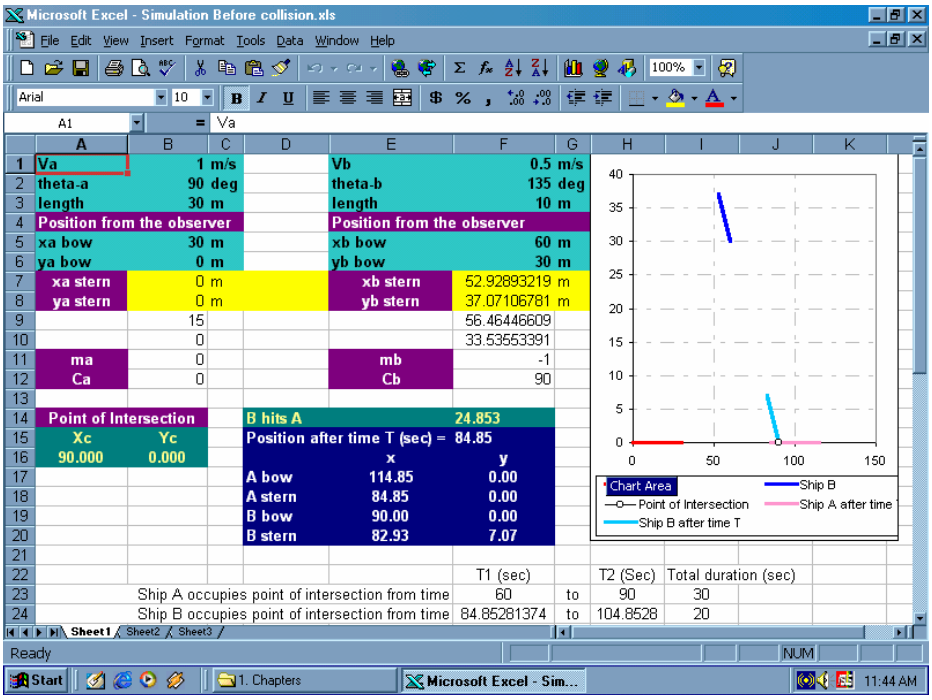


Figure 3.10: Developed computer program for analysis before collision.

At the end of this part of the problem it may be possible to summarise the answers of the questions that were put forward at the beginning of the section:

Table 3.1: Summary of simulation before collision

1	With the given speeds and headings, will there be a collision between two ships?	If the following relations are satisfied (a) $TA_{arrival} < TB_{arrival} < TA_{departure}$ Or (b) $TB_{arrival} < TA_{arrival} < TB_{departure}$
2	If there is a collision, where will it take place (i.e the location)?	At point C (Xc, Yc)
3	Which ship strikes the other?	For 1 (a), Ship B hits Ship A For 1 (b), Ship A hits Ship B
4	At which location the bodies get hit?	H units abaft bow of the struck ship

3.3.2 Model for Simulation During and After Collision

At this stage it may be assumed that two ships ply on their in own such a way that at some point in their path after sailing some distances, both the ships collide with each other. In most of the cases, where bow of one of the ships hits the other vessel, the first ship is named the “Striking Ship” and the other ship which is being hit at any place excluding its bow is called the “Struck Ship”. In the case of a head on collision or a collision at the bows (if angle is less than 180 degree), the vessel with relatively lesser forward speed than the other is considered as the “Struck Ship” while the ship with relatively greater forward speed is being tagged as "Striking Ship". The zone on the water surface where the collision takes place may be called "Collision Zone".

By considering a collision scenario, as shown in Figure 3.11, where Ship A strikes Ship B two co-ordinate systems are being assumed for each ship such as X-Y for striking ship and I-J for struck ship.

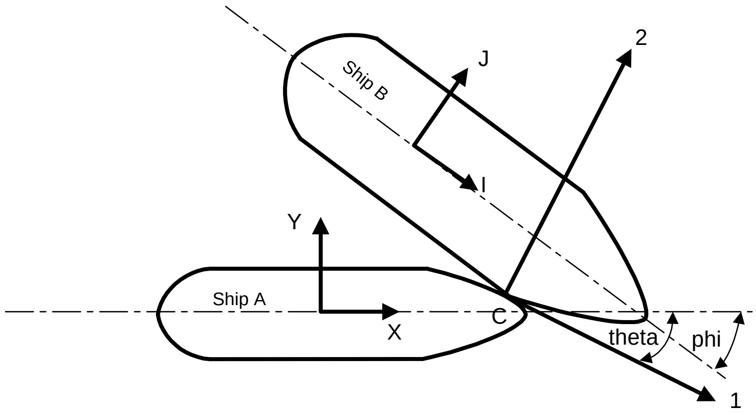


Figure 3.11: Co-ordinate system of a ship-ship collision.

The following notations will be followed for the development of the model:

c = Point of impact at the collision surface

1 = Direction tangent at the point of impact

2 = Direction normal to 1-axis

X = Direction along centreline of the striking vessel

Y = Direction along the transverse axis of striking vessel

I = Direction along centreline of the struck vessel

J = Direction along the transverse axis of struck vessel

θ = Angle between X axis and 1 axis

φ = Angle between X axis and I axis

M_A = Mass of ship A

M_B = Mass of ship B

V_A = Forward velocity of Ship A

V_B = Forward velocity of Ship B

$V_{A1\ after}$ = Velocity of ship A in the direction of 1-axis after collision

$V_{A1\ before}$ = Velocity of ship A in the direction of 1-axis before collision

$V_{A2\ after}$ = Velocity of ship A in the direction of 2-axis after collision

$V_{A2\ before}$ = Velocity of ship A in the direction of 2-axis before collision

$V_{B1\ after}$ = Velocity of ship B in the direction of 1-axis after collision

$V_{B1\ before}$ = Velocity of ship B in the direction of 1-axis before collision

$V_{B2\ after}$ = Velocity of ship B in the direction of 2-axis after collision

$V_{B2\ before}$ = Velocity of ship B in the direction of 2-axis before collision

V_{A1} = V_{B1} = Common velocity in the direction of 1-axis after reaching maximum pressure

V_{A2} = V_{B2} = Common velocity in the direction of 2-axis after reaching maximum pressure

E = Co-efficient of restitution, varies in between 0 (perfectly inelastic) to 1 (perfectly elastic)

T_{col} = Collision time, the time required to change the momentum or velocities

Fn = Force in n-axis direction (e.g. FX is force in the direction of X-axis)

3.3.2.1 The Collision Forces

Forces on Ship A in X-axis and Y-axis direction

$$F_X = F_1 \cos\theta + F_2 \cos(90-\theta)$$
$$F_Y = - F_1 \sin\theta + F_2 \sin(90-\theta)$$

Forces on Ship B in I-axis and J-axis direction

$$F_I = F_1 \cos(\varphi-\theta) + F_2 \cos(\varphi-\theta)$$
$$F_J = F_1 \cos(\varphi-\theta) + F_2 \cos(\varphi-\theta)$$

Forces F_1 and F_2 are perpendicular forces acting at the contact point *C* due to impact of the two bodies. It is known that impact force at a particular direction is equal to change of linear momentum in that direction, i.e.

F_1 = Change in momentum in 1-axis direction

F_2 = Change in momentum in 2-axis direction

Let us consider the following for both the ships before collision,

For Ship A	For Ship B
$V_{A1\ before} = V_A \cos\theta$	$V_{B1\ before} = V_B \cos(\varphi-\theta)$
$V_{A2\ before} = V_A \sin\theta$	$V_{B2\ before} = V_B \sin(\varphi-\theta)$

Therefore, by using above expressions the forces may be obtained, For Ship A,

$$F_{A1} = M_A \frac{V_{A1after} - V_{A1before}}{T_{col}} \qquad F_{A2} = M_A \frac{V_{A2after} - V_{A2before}}{T_{col}}$$

For Ship B,

$$F_{B1} = M_B \frac{V_{B1after} - V_{B1before}}{T_{col}} \qquad F_{B2} = M_B \frac{V_{B2after} - V_{B2before}}{T_{col}}$$

Where, F_{A1} = Force on Ship A in the direction of 1-Axis

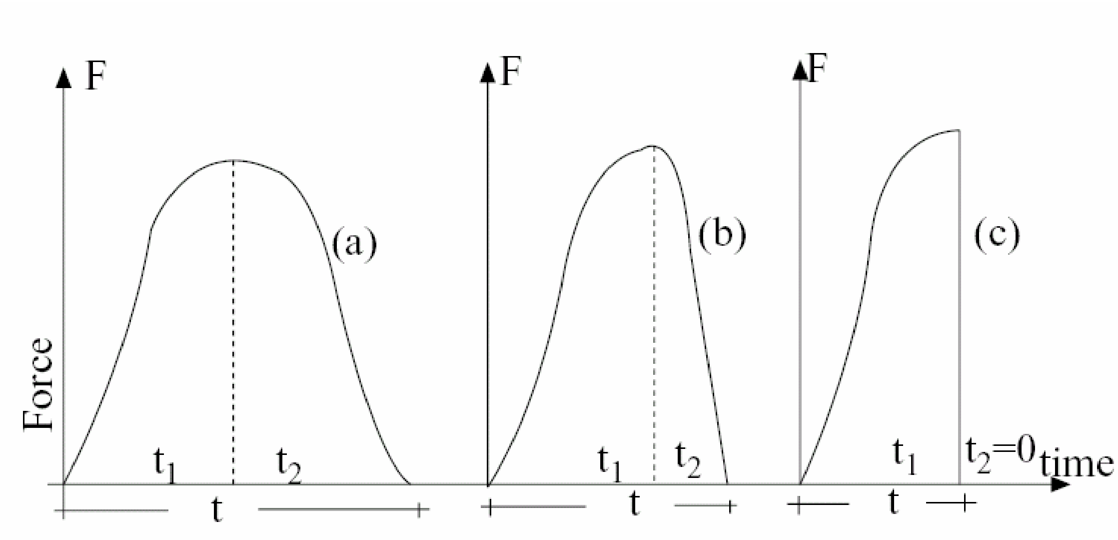
F_{A2} = Force on Ship A in the direction of 2-Axis

F_{B1} = Force on Ship B in the direction of 1-Axis

F_{B2} = Force on Ship B in the direction of 2-Axis

3.3.2.2 The Coefficient of Restitution (E)

The coefficient of restitution is a measure of the elasticity of the collision between two objects. Elasticity is a measure of how much of the kinetic energy of the colliding objects before the collision remains as kinetic energy of the objects after the collision. With an inelastic collision, some kinetic energy is transformed into deformation of the material, heat, sound, and other forms of energy. A perfectly elastic collision has a coefficient of restitution of 1. Example: two diamonds bouncing off each other. A perfectly plastic, or inelastic, collision has E = 0. Example: two lumps of clay that don't bounces at all, but stick together. So the coefficient of restitution will always be between zero and one. Oztas [40] discussed elaborately the application of coefficient of restitution with respect to car crashes. In the analysis Oztas divided the impact Time t into t_1 and t_2, as shown in Figure 3.12. Where t_1 is the compression time and, during this time, the object starts to change its shape and reaches its maximum level, i.e., Force and Reshaping are at a maximum; after this time, the relative velocity between the objects is zero. Again t_2 is the restitution time and simultaneously the reshaping lessens and finally disappears.



(a) (b) (c)

Figure 3.12: Relations between Force and Time during impact [40].

In the first diagram, $t_1 = t_2 = 1/2t$; it corresponds to an Elastic Collision as referred to in Fig. 3.12a. In the second diagram, $t_1 \neq t_2$, and this case is Inelastic, meaning that it is an Elastoplastic Collision (Figure 3.12b). In the third diagram, t_2 becomes zero ($t_2 = 0$), that means the objects collide and cannot be separated. This impact is a Plastic Collision (Figure 3.12c).

Now when a collision starts taking place the change in momentum is equal to the impulse integral and the common velocity at the beginning of the restitution time reaches the maximum level. Considering Figure 3.11 and 3.12, the following mathematical relations are obtained.

	For Ship A	For Ship B
Along 1-Axis	$M_A\left(V_{A1} - V_{A1before}\right) = \int_0^{t_1} F_{B1}dt$	$M_B\left(V_{B1} - V_{B1before}\right) = \int_0^{t_1} F_{A1}dt$
Along 2-Axis	$M_A\left(V_{A2} - V_{A2before}\right) = \int_0^{t_1} F_{B2}dt$	$M_B\left(V_{B2} - V_{B2before}\right) = \int_0^{t_1} F_{A2}dt$

According to impulse momentum theory,

Along 1-Axis $$\int_0^{t_1} F_{A1} dt + \int_0^{t_1} F_{B1} dt = 0$$

Along 2-Axis $$\int_0^{t_1} F_{A2} dt + \int_0^{t_1} F_{B2} dt = 0$$

Thus the common velocities are obtained,

Along 1-Axis $$V_{A1} = V_{B1} = \frac{M_B V_{B1\,before} + M_A V_{A1\,before}}{M_B + M_A}$$

Along 2-Axis $$V_{A2} = V_{B2} = \frac{M_B V_{B2\,before} + M_A V_{A2\,before}}{M_B + M_A}$$

During the restitution time (t2) the followings are obtained,

	For Ship A	For Ship B
Along 1-Axis	$M_A\left(V_{A1after} - V_{A1}\right) = \int_{t_1}^{t_2} F_{B1} dt$	$M_B\left(V_{B1after} - V_{B1}\right) = \int_{t_1}^{t_2} F_{A1} dt$
Along 2-Axis	$M_A\left(V_{A2after} - V_{A2}\right) = \int_{t_1}^{t_2} F_{B2} dt$	$M_B\left(V_{B2after} - V_{B2}\right) = \int_{t_1}^{t_2} F_{A2} dt$

According to impulse momentum theory,

Along 1-Axis $$\int_{t_1}^{t_2} F_{A1} dt + \int_{t_1}^{t_2} F_{B1} dt = 0$$

Along 2-Axis $$\int_{t_1}^{t_2} F_{A2} dt + \int_{t_1}^{t_2} F_{B2} dt = 0$$

Thus the common velocities are,

Along 1-Axis $$V_{A1} = V_{B1} = \frac{M_B V_{B1\,after} + M_A V_{A1\,after}}{M_B + M_A}$$

Along 2-Axis $V_{A2} = V_{B2} = \dfrac{M_B V_{B2\ after} + M_A V_{A2\ after}}{M_B + M_A}$

It is now possible to establish a relationship between impulse integrals with the help of Coefficient of Restitution (*E*). This relation can be expressed as the following:

$$\int_{t_1}^{t_2} F dt = E \int_{0}^{t_1} F dt = 0$$

According to Oztas (1999), the Coefficient of Restitution (E) is related to objects, materials, masses and velocities. Considering Figure 3.13, the following parameters are important to note.

A1 = Area of Compression period.

A2 = Area of Restitution period.

A2/A1 = E

If *E* = 1, the impact is Elastic.

If *E* = 0 to 1, the impact is Elastoplastic.

If *E* = 0, the impact is Plastic.

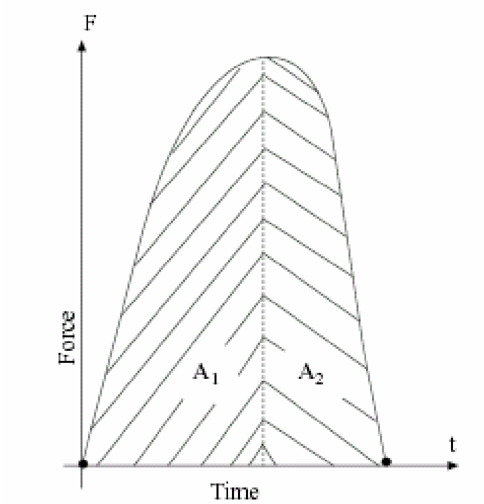


Figure 3.13: Relation between force and time [40]

Therefore, it is now possible to obtain the expressions of velocities after collision for both the ships using the impulse integral. Such as,

For Ship A: $V_{A1after} = V_{A1}(1+E) - E \times V_{A1before}$ & $V_{A2after} = V_{A2}(1+E) - E \times V_{A2before}$

For Ship B: $V_{B1after} = V_{B1}(1+E) - E \times V_{B1before}$ & $V_{B2after} = V_{B2}(1+E) - E \times V_{B2before}$

The loss of kinetic energy is therefore, obtained according to the following:

	For Ship A	For Ship B
Along 1-Axis:	$KE_{A1} = \frac{1}{2} M_A \left(V_{A1before}{}^2 - V_{A1after}{}^2 \right)$	$KE_{B1} = \frac{1}{2} M_B \left(V_{B1before}{}^2 - V_{B1after}{}^2 \right)$
Along 2-Axis:	$KE_{A2} = \frac{1}{2} M_A \left(V_{A2before}{}^2 - V_{A2after}{}^2 \right)$	$KE_{B2} = \frac{1}{2} M_B \left(V_{B2before}{}^2 - V_{B2after}{}^2 \right)$

Using the expressions of velocities before and after collision, the forces on the struck ship may be computed according to the following:

Force on Ship A in X-axis direction (surge direction)

$$F_X = F_1 \cos\theta + F_2 \cos(90-\theta)$$

$$= M_A \frac{V_{A1after} - V_{A1before}}{T} \cos\theta + M_A \frac{V_{A2after} - V_{A2before}}{T} \cos(90-\theta)$$

$$= M_A \frac{\left\{ V_{AI}(1+E) - EV_{A1before} \right\} - V_{A1before}}{T} \cos\theta +$$

$$M_A \frac{\left\{ V_{AJ}(1+E) - EV_{A2before} \right\} - V_{A2before}}{T} \cos(90-\theta)$$

$$= M_A \frac{\left\{ \frac{M_B V_{B1before} + M_A V_{A1before}}{M_A + M_B} (1+E) - EV_{A1before} \right\} - V_{A1before}}{T} \cos\theta +$$

$$M_A \frac{\left\{ \frac{M_B V_{B2before} + M_A V_{A2before}}{M_A + M_B} (1+E) - EV_{A2before} \right\} - V_{A2before}}{T} \cos(90-\theta)$$

$$= M_A \frac{\left\{\frac{M_B V_B \cos(\varphi-\theta)+M_A V_A \cos\theta}{M_A+M_B}(1+E)-EV_A\cos\theta\right\}-V_A\cos\theta}{T}\cos\theta +$$

$$M_A \frac{\left\{\frac{M_B V_B \sin(\varphi-\theta)+M_A V_A \sin\theta}{M_A+M_B}(1+E)-EV_A\sin\theta\right\}-V_A\sin\theta}{T}\cos(90-\theta)$$

Force on Ship A in Y-axis direction (sway direction)

$$F_Y = F_1 \sin\theta + F_2 \sin(90-\theta)$$

$$= M_A \frac{V_{A1after} - V_{A1before}}{T}\sin\theta + M_A \frac{V_{A2after} - V_{A2before}}{T}\sin(90-\theta)$$

$$= M_A \frac{\{V_{AI}(1+E)-EV_{A1before}\}-V_{A1before}}{T}\sin\theta +$$

$$M_A \frac{\{V_{AJ}(1+E)-EV_{A2before}\}-V_{A2before}}{T}\sin(90-\theta)$$

$$= M_A \frac{\left\{\frac{M_B V_{B1before}+M_A V_{A1before}}{M_A+M_B}(1+E)-EV_{A1before}\right\}-V_{A1before}}{T}\sin\theta +$$

$$M_A \frac{\left\{\frac{M_B V_{B2before}+M_A V_{A2before}}{M_A+M_B}(1+E)-EV_{A2before}\right\}-V_{A2before}}{T}\sin(90-\theta)$$

$$= M_A \frac{\left\{\frac{M_B V_B \cos(\varphi-\theta)+M_A V_A \cos\theta}{M_A+M_B}(1+E)-EV_A\cos\theta\right\}-V_A\cos\theta}{T}\sin\theta +$$

$$M_A \frac{\left\{\frac{M_B V_B \sin(\varphi-\theta)+M_A V_A \sin\theta}{M_A+M_B}(1+E)-EV_A\sin\theta\right\}-V_A\sin\theta}{T}\sin(90-\theta)$$

Force on Ship B in I-axis direction (surge direction)

$$F_I = F_1 \cos(\varphi-\theta) + F_2 \cos(90-\varphi+\theta)$$

$$= M_B \frac{V_{B1after} - V_{B1before}}{T} \cos(\varphi-\theta) + M_B \frac{V_{B2after} - V_{B2before}}{T} \cos(90-\varphi+\theta)$$

$$= M_B \frac{\{V_{BI}(1+E) - EV_{B1before}\} - V_{B1before}}{T} \cos(\varphi-\theta) +$$
$$M_B \frac{\{V_{BJ}(1+E) - EV_{B2before}\} - V_{B2before}}{T} \cos(90-\varphi+\theta)$$

$$= M_B \frac{\left\{\frac{M_B V_{B1before} + M_A V_{A1before}}{M_A + M_B}(1+E) - EV_{B1before}\right\} - V_{B1before}}{T} \cos(\varphi-\theta) +$$
$$M_B \frac{\left\{\frac{M_B V_{B2before} + M_A V_{A2before}}{M_A + M_B}(1+E) - EV_{B2before}\right\} - V_{B2before}}{T} \cos(90-\varphi+\theta)$$

$$= M_B \frac{\left\{\frac{M_B V_B \cos(\varphi-\theta) + M_A V_A \cos\theta}{M_A + M_B}(1+E) - EV_B \cos(\varphi-\theta)\right\} - V_B \cos(\varphi-\theta)}{T} \cos\theta +$$
$$M_B \frac{\left\{\frac{M_B V_B \sin(\varphi-\theta) + M_A V_A \sin\theta}{M_A + M_B}(1+E) - EV_B \sin(\varphi-\theta)\right\} - V_B \sin(\varphi-\theta)}{T} \cos(90-\theta)$$

Force on Ship B in J-axis direction (sway direction)

$$F_J = F_1 \sin(\varphi-\theta) + F_2 \sin(90-\varphi+\theta)$$

$$= M_B \frac{V_{B1after} - V_{B1before}}{T} \sin(\varphi-\theta) + M_B \frac{V_{B2after} - V_{B2before}}{T} \sin(90-\varphi+\theta)$$

$$= M_B \frac{\{V_{BI}(1+E) - EV_{B1before}\} - V_{B1before}}{T} \sin(\varphi-\theta) +$$
$$M_B \frac{\{V_{BJ}(1+E) - EV_{B2before}\} - V_{B2before}}{T} \sin(90-\varphi+\theta)$$

$$= M_B \frac{\left\{ \frac{M_B V_{B1before} + M_A V_{A1before}}{M_A + M_B}(1+E) - EV_{B1before} \right\} - V_{B1before}}{T} \sin(\varphi - \theta) +$$
$$M_B \frac{\left\{ \frac{M_B V_{B2before} + M_A V_{A2before}}{M_A + M_B}(1+E) - EV_{B2before} \right\} - V_{B2before}}{T} \sin(90 - \varphi + \theta)$$

$$= M_B \frac{\left\{ \frac{M_B V_B \cos(\varphi - \theta) + M_A V_A \cos\theta}{M_A + M_B}(1+E) - EV_B \cos(\varphi - \theta) \right\} - V_B \cos(\varphi - \theta)}{T} \sin\theta +$$
$$M_B \frac{\left\{ \frac{M_B V_B \sin(\varphi - \theta) + M_A V_A \sin\theta}{M_A + M_B}(1+E) - EV_B \sin(\varphi - \theta) \right\} - V_B \sin(\varphi - \theta)}{T} \sin(90 - \theta)$$

These forces will however create moments that will all together impel the ships in two translating and three rotational motions, namely surge & sway and roll, pitch & yaw, as shown in the Figure 3.14.

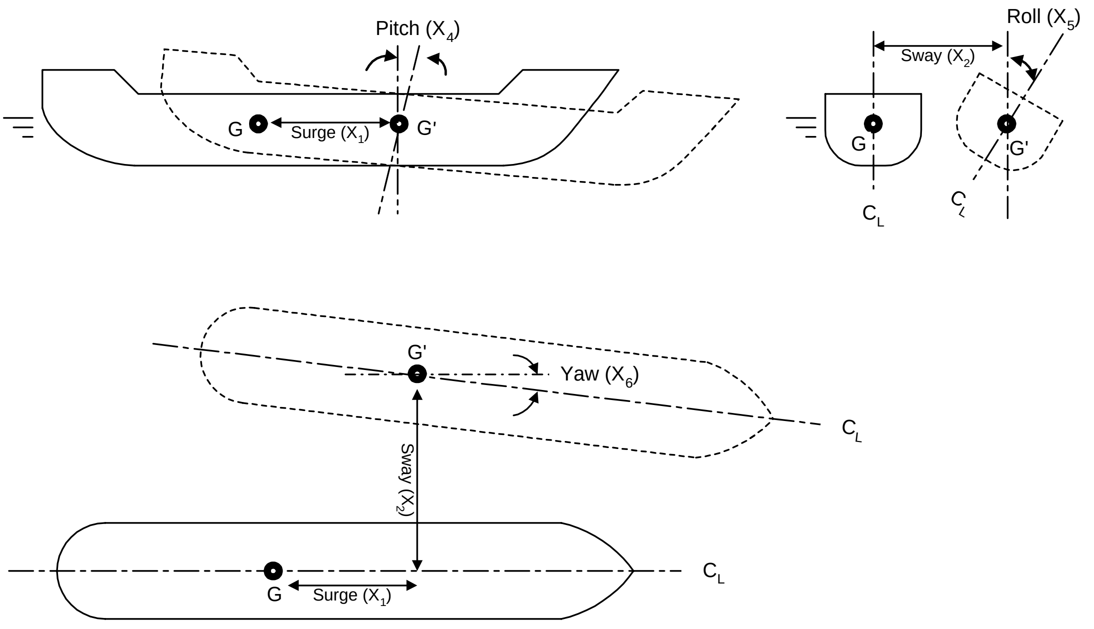


Figure 3.14: Translating and rotational motions of a ship due to external forces and moments

3.4 Solution of the Equation of Motion and Characteristics of the Force

The equation of motion needs to be solved with necessary boundary conditions in order to find the ships responses due to collision forces. During a collision the equation of motion may be expressed as the following,

$$M_V \frac{d^2x_i}{dt^2} + b_{ij}\frac{dx_i}{dt} + c_{ij}x_i = Fc_i(t)$$

$$\Rightarrow \frac{d^2x_i}{dt^2} + \frac{b_{ij}}{M_V}\frac{dx_i}{dt} + \frac{c_{ij}}{M_V}x_i = \frac{Fc_i(t)}{M_V}$$

Where,

$Fc_i(t)$ = Force/Moment due to collision in *i* direction

M_V = Ships Virtual mass/virtual mass moment of inertia

b_{ij} & c_{ij} = Damping and restoring force coefficients respectively

ij = Motion in *i* direction due to force/moment in *j* direction

However, the collision force term in the above equation acts only during the collision period. This term remains zero before contact and also after contact between the two vessels. Thus the equation of motion without collision force becomes,

$$M_V \frac{d^2x_i}{dt^2} + b_{ij}\frac{dx_i}{dt} + c_{ij}x_i = 0$$

$$\Rightarrow \frac{d^2x_i}{dt^2} + \frac{b_{ij}}{M_V}\frac{dx_i}{dt} + \frac{c_{ij}}{M_V}x_i = 0$$

Where, the notations represent the same meanings as the above equation of motion with collision force.

The auxiliary equation for the motion equation may be written using the operator D which can be treated as algebraic quantity,

$$D^2 + \frac{b}{M_V} D + \frac{c}{M_V} = 0$$

or, $$D = -\frac{b}{2M_V} \pm i\sqrt{\frac{c}{M_V} - \left(\frac{b}{2M_V}\right)^2}$$

or, $$D = \alpha \pm i\beta$$

Where,

$$\alpha = -\frac{b}{2M_V} \quad \text{and} \quad \beta = \sqrt{\frac{c}{M_V} - \left(\frac{b}{2M_V}\right)^2}$$

Therefore, the general solution of the equation may be expressed as,

$$x_i = e^{\alpha t}\left\{A_1 \sin \beta t + A_2 \cos \beta t\right\}$$

Where, A_1 and A_2 are constants which are needed to be determined using appropriate boundary conditions.

Assuming an initial condition when the collision force is maximum at time $t = t_{max} = 0$, the displacement is $x_i = x_{i0}$. According to the theory of simple harmonic motion, this amplitude or displacement is maximum when the velocity reaches to zero and the velocity becomes maximum when the amplitude becomes zero units. Therefore, assuming x_{i0} is the maximum amplitude due to collision force at the time $t = 0$, the following unknowns are obtained from the equation of general solution as derived above.

$$A_1 = -\frac{x_{i0}}{\beta} \quad \& \quad A_2 = x_{i0}$$

And therefore, the general solution becomes,

$$x_i = x_{i0} e^{\alpha t} \left\{ \cos \beta t - \frac{1}{\beta} \sin \beta t \right\}$$

The above equation is similar to the damping part of any equation of motion where x_{i0} resembles the maximum amplitude due to an excitation and $e^{\alpha t}$ resembles exponential decay of the motion. In order to obtain the maximum amplitude, it is required that the particular integral of the force term *Fc(t)* of the equation of motion has to be solved. However, the function *Fc(t)* is generally considered very complex to predict as the complicated internal structural arrangement, including the external fenders, of ship hull are subject to progressive structural deformations/failures by buckling, shearing, tearing, crushing, bending and twisting of fenders, plates, stringers, panels etc during a collision.

In this study, several collision force functions are proposed. In the first case the collision force is considered as a constant variable which doesn't change with the time during the collision and in the other cases, the collision forces are considered as a variable function of time *(t).* Such cases are:

1. The force is constant over the impact period, $Fc = F_{\max}$
2. Linear function of time, $Fc_i(t) = F_{\max i} f(t)$
3. Trigonometric function of time, $Fc_i(t) = F_{\max i} f(\sin t)$
4. Exponential function of time, $Fc_i(t) = F_{\max i} f(e^t)$

3.4.1 Constant Force Over the Impact Period, $Fc = F_{\max}$

In this case it is assumed that the collision force is constant over the contact period (in between t_{sep} and t_{hit}) of the two vessels. The collision forces are zero before and after collision. The following figure reveals the condition elaborately.

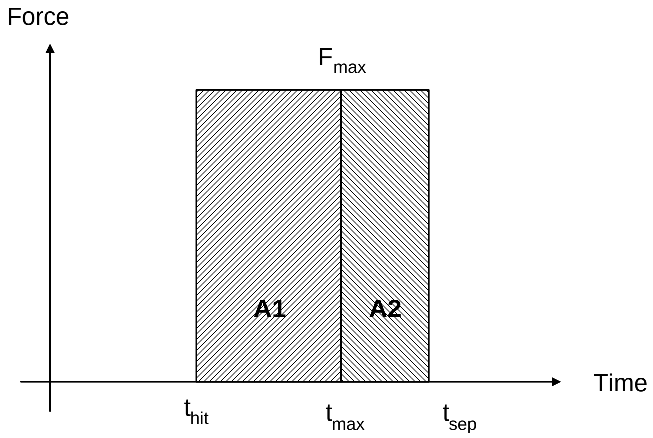


Figure 3.15: Collision force is constant over the contact period.

The conditions for collision force along the time line are expressed as follows:

For	t= 0 to t_{hit}	$F = 0$
	t = t_{hit} to t = t_{max}	$F = F_{\max}$
	t = t_{max} to t = t_{sep}	$F = F_{\max}$
	t= t_{sep} to t = $\propto$	$F = 0$

The particular integral is required to be obtained by assuming an Euler's number of power zero multiplied to the right hand side of the equation of motion and thereafter operating the expression by replacing D with zero, which is at the auxiliary equation operating as denominator. That is,

$$x_i = \frac{\frac{F_{\max}}{a_{ij}}}{D^2 + \frac{b_{ij}}{a_{ij}} D + \frac{c_{ij}}{a_{ij}}} \Rightarrow x_i = \frac{\frac{F_{\max}}{a_{ij}} e^{0\times t}}{D^2 + \frac{b_{ij}}{a_{ij}} D + \frac{c_{ij}}{a_{ij}}} \Rightarrow x_i = \frac{\frac{F_{\max}}{a_{ij}} e^{0\times t}}{0^2 + \frac{b_{ij}}{a_{ij}} \times 0 + \frac{c_{ij}}{a_{ij}}} \therefore x_i = \frac{\frac{F_{\max}}{a_{ij}}}{\frac{c_{ij}}{a_{ij}}} = \frac{F_{\max}}{c_{ij}}$$

3.4.2 Force as a Linear Function of Time $Fc_i(t) = F_{\max i} f(t)$

In this case the force is assumed to be linear functions of time. That is the force increases during the compression period ($t_{hit} < t < t_{max}$) reaching up to the maximum vale F_{max} at time t_{max} and then decreasing linearly until the vessels separate at time $t = t_{sep}$. The following figure reveals the case.

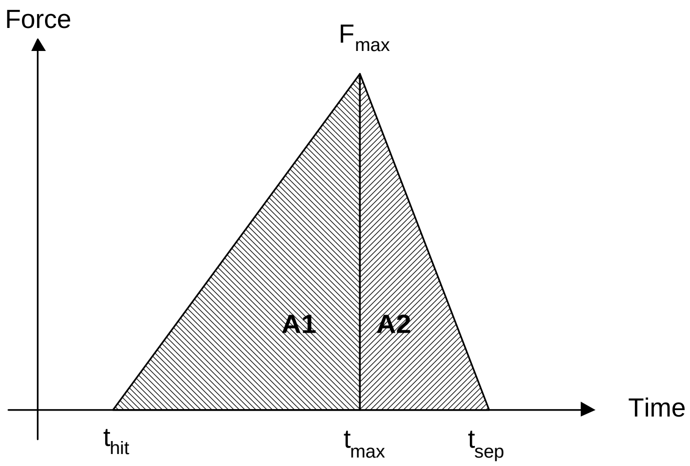


Figure 3.16: Collision force is a linear function over the contact period.

The conditions for collision force along the time line are expressed as follows:

For t= 0 to t_{hit} $\quad F = 0$

t = t_{hit} to t = t_{max} $\quad F = \dfrac{F_{\max}}{t_{\max} - t_{hit}}\left(t - t_{hit}\right)$

t = t_{max} to t = t_{sep} $\quad F = \dfrac{F_{\max}}{t_{sep} - t_{\max t}}\left(t_{sep} - t\right)$

t= t_{sep} to t = $\propto$ $\quad F = 0$

The particular integral is obtained by operating the force terms with the auxiliary equation in the denominator. The auxiliary equation is expressed as an infinite series and in this particular case D works as a differential operator.

For $t = t_{hit}$ to $t = t_{max}$

$$x_i = \frac{\dfrac{F_{\max}}{a_{ij}\left(t_{\max}-t_{hit}\right)}\left(t-t_{hit}\right)}{D^2+\dfrac{b_{ij}}{a_{ij}}D+\dfrac{c_{ij}}{a_{ij}}}$$

$$\Rightarrow x_i = \frac{\dfrac{F_{\max}}{a_{ij}\left(t_{\max}-t_{hit}\right)}\left(t-t_{hit}\right)}{\dfrac{c_{ij}}{a_{ij}}\left(1+\dfrac{b_{ij}}{c_{ij}}D+\dfrac{a_{ij}}{c_{ij}}D^2\right)}$$

$$\Rightarrow x_i = \frac{F_{\max}}{a_{ij}\left(t_{\max}-t_{hit}\right)}\frac{a_{ij}}{c_{ij}}\left(1+\frac{b_{ij}}{c_{ij}}D+\frac{a_{ij}}{c_{ij}}D^2\right)^{-1}\left(t-t_{hit}\right)$$

$$\Rightarrow x_i = \frac{F_{\max}}{c_{ij}\left(t_{\max}-t_{hit}\right)}\left(1-\frac{b_{ij}}{c_{ij}}D+\ldots\right)\left(t-t_{hit}\right)$$

$$\therefore x_i = \frac{F_{\max}}{c_{ij}\left(t_{\max}-t_{hit}\right)}\left[\left(t-t_{hit}\right)-\frac{b_{ij}}{c_{ij}}\right]$$

For, $t = t_{max}$ to $t = t_{sep}$

$$x_i = \frac{\dfrac{F_{\max}}{a_{ij}\left(t_{sep}-t_{\max}\right)}\left(t_{sep}-t\right)}{D^2+\dfrac{b_{ij}}{a_{ij}}D+\dfrac{c_{ij}}{a_{ij}}}$$

$$\Rightarrow x_i = \frac{\dfrac{F_{\max}}{a_{ij}\left(t_{sep}-t_{\max}\right)}\left(t_{sep}-t\right)}{\dfrac{c_{ij}}{a_{ij}}\left(1+\dfrac{b_{ij}}{c_{ij}}D+\dfrac{a_{ij}}{c_{ij}}D^2\right)}$$

$$\Rightarrow x_i = \frac{F_{\max}}{a_{ij}\left(t_{\max}-t_{hit}\right)}\frac{a_{ij}}{c_{ij}}\left(1+\frac{b_{ij}}{c_{ij}}D+\frac{a_{ij}}{c_{ij}}D^2\right)^{-1}\left(t_{sep}-t\right)$$

$$\Rightarrow x_i = \frac{F_{\max}}{c_{ij}\left(t_{sep}-t_{\max}\right)}\left(1-\frac{b_{ij}}{c_{ij}}D+\ldots\right)\left(t_{sep}-t\right)$$

$$\therefore x_i = \frac{F_{\max}}{c_{ij}\left(t_{sep}-t_{\max}\right)}\left[\left(t_{sep}-t\right)+\frac{b_{ij}}{c_{ij}}\right]$$

3.4.3 Force as a Trigonometric Function of Time, $Fc_i(t) = F_{\max i} f(\sin t)$

In this particular case the force is assumed to change as a trigonometric function of time. The force is expressed as sine functions where the force value is maximum at $t = t_{max}$ and minimum (zero) at $t = t_{hit}$ and $t = t_{sep}$.

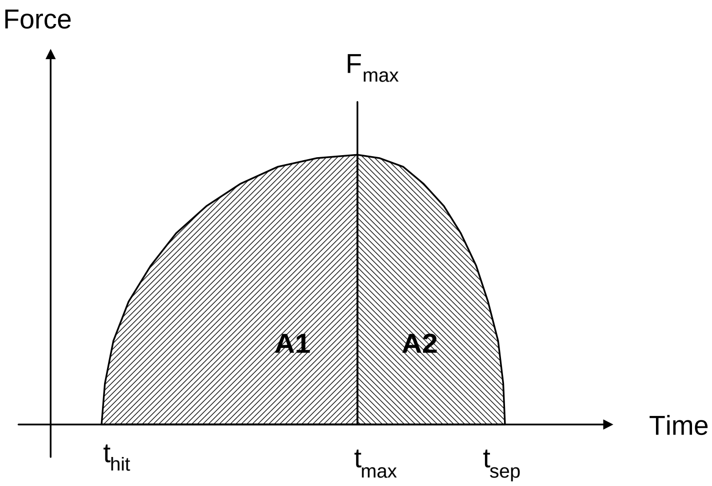


Figure 3.17: Collision force is a trigonometric function over the contact period.

The conditions for collision force along the time line are expressed as follows:

For $t = 0$ to t_{hit} $\quad F = 0$

$t = t_{hit}$ to $t = t_{max}$ $\quad F = F_{\max} \sin\left[\frac{\pi}{2}\left(\frac{t - t_{hit}}{t_{\max} - t_{hit}}\right)\right]$

$t = t_{max}$ to $t = t_{sep}$ $\quad F = F_{\max} \sin\left[\frac{\pi}{2}\left(\frac{t_{sep} - t}{t_{sep} - t_{\max}}\right)\right]$

$t = t_{sep}$ to $t = \propto$ $\quad F = 0$

The particular integral is obtained by operating the right hand side of the motion equation with the auxiliary equation in the denominator. In the denominator the operator D^2 is replaced by the constant coefficient of the variable time t and a negative sign is being put in front. The remaining *D* operators are worked out as the differential operators.

For $t = t_{hit}$ to $t = t_{max}$

$$x_i = \frac{\frac{F_{\max}}{a_{ij}} \sin\left[\frac{\pi}{2}\left(\frac{t - t_{hit}}{t_{\max} - t_{hit}}\right)\right]}{D^2 + \frac{b_{ij}}{a_{ij}} D + \frac{c_{ij}}{a_{ij}}}$$

$$\Rightarrow x_i = \frac{\frac{F_{\max}}{a_{ij}} \sin\left[\frac{\pi}{2(t_{\max} - t_{hit})} t - \frac{\pi t_{hit}}{2(t_{\max} - t_{hit})}\right]}{D^2 + \frac{b_{ij}}{a_{ij}} D + \frac{c_{ij}}{a_{ij}}}$$

$$\Rightarrow x_i = \frac{\frac{F_{\max}}{a_{ij}} \sin\left[\frac{\pi}{2(t_{\max} - t_{hit})} t - \frac{\pi t_{hit}}{2(t_{\max} - t_{hit})}\right]}{-\left\{\frac{\pi}{2(t_{\max} - t_{hit})}\right\}^2 + \frac{b_{ij}}{a_{ij}} D + \frac{c_{ij}}{a_{ij}}}$$

$$\Rightarrow x_i = \frac{\frac{F_{\max}}{a_{ij}} \sin\left[\frac{\pi}{2(t_{\max} - t_{hit})} t - \frac{\pi t_{hit}}{2(t_{\max} - t_{hit})}\right]}{\frac{c_{ij}}{a_{ij}}\left[\frac{b_{ij}}{c_{ij}} D + \left\{1 - \frac{a_{ij}\pi^2}{4c_{ij}(t_{\max} - t_{hit})^2}\right\}\right]}$$

$$\Rightarrow x_i = \frac{F_{\max}\left[\sin\left\{\frac{\pi}{2(t_{\max} - t_{hit})} t - \frac{\pi t_{hit}}{2(t_{\max} - t_{hit})}\right\}\right] \times \left[\frac{b_{ij}}{c_{ij}} D + \left\{1 - \frac{a_{ij}\pi^2}{4c_{ij}(t_{\max} - t_{hit})^2}\right\}\right]}{c_{ij}\left[\frac{b_{ij}}{c_{ij}} D + \left\{1 - \frac{a_{ij}\pi^2}{4c_{ij}(t_{\max} - t_{hit})^2}\right\}\right]\left[\frac{b_{ij}}{c_{ij}} D - \left\{1 - \frac{a_{ij}\pi^2}{4c_{ij}(t_{\max} - t_{hit})^2}\right\}\right]}$$

$$\Rightarrow x_i = \frac{\frac{b_{ij}}{c_{ij}} D\left[F_{\max} \sin\left\{\frac{\pi}{2(t_{\max} - t_{hit})} t - \frac{\pi t_{hit}}{2(t_{\max} - t_{hit})}\right\}\right]}{c_{ij}\left[\frac{b_{ij}^2}{c_{ij}^2} D^2 - \left\{1 - \frac{a_{ij}\pi^2}{4c_{ij}(t_{\max} - t_{hit})^2}\right\}^2\right]} +$$

$$\frac{F_{\max}\left[\sin\left\{\frac{\pi}{2(t_{\max} - t_{hit})} t - \frac{\pi t_{hit}}{2(t_{\max} - t_{hit})}\right\}\right] \times \left\{1 - \frac{a_{ij}\pi^2}{4c_{ij}(t_{\max} - t_{hit})^2}\right\}}{c_{ij}\left[\frac{b_{ij}^2}{c_{ij}^2} D^2 - \left\{1 - \frac{a_{ij}\pi^2}{4c_{ij}(t_{\max} - t_{hit})^2}\right\}^2\right]}$$

$$\therefore x_i = \frac{\frac{b_{ij}}{c_{ij}}\frac{2(t_{\max}-t_{hit})}{\pi}F_{\max}\cos\left\{\frac{\pi}{2(t_{\max}-t_{hit})}t-\frac{\pi t_{hit}}{2(t_{\max}-t_{hit})}\right\}}{c_{ij}\left[-\frac{b_{ij}^2}{c_{ij}^2}\left\{\frac{\pi}{2(t_{\max}-t_{hit})}\right\}^2-\left\{1-\frac{a_{ij}\pi^2}{4c_{ij}(t_{\max}-t_{hit})^2}\right\}^2\right]}+$$

$$\frac{F_{\max}\left\{1-\frac{a_{ij}\pi^2}{4c_{ij}(t_{\max}-t_{hit})^2}\right\}\sin\left\{\frac{\pi}{2(t_{\max}-t_{hit})}t-\frac{\pi t_{hit}}{2(t_{\max}-t_{hit})}\right\}}{c_{ij}\left[-\frac{b_{ij}^2}{c_{ij}^2}\left\{\frac{\pi}{2(t_{\max}-t_{hit})}\right\}^2-\left\{1-\frac{a_{ij}\pi^2}{4c_{ij}(t_{\max}-t_{hit})^2}\right\}^2\right]}$$

Putting, $\frac{\pi}{2(t_{\max}-t_{hit})}=K_{Tb}$ the particular integral may be obtained as the following,

$$\therefore x_i = \frac{\frac{b_{ij}}{c_{ij}}\frac{1}{K_{Tb}}F_{\max}\cos\{K_{Tb}t-K_{Tb}t_{hit}\}}{c_{ij}\left[-\frac{b_{ij}^2}{c_{ij}^2}K_{Tb}^2-\left\{1-\frac{a_{ij}}{c_{ij}}K_{Tb}^2\right\}^2\right]}+\frac{F_{\max}\left\{1-\frac{a_{ij}}{c_{ij}}K_{Tb}^2\right\}\sin\{K_{Tb}t-K_{Tb}t_{hit}\}}{c_{ij}\left[-\frac{b_{ij}^2}{c_{ij}^2}K_{Tb}^2-\left\{1-\frac{a_{ij}}{c_{ij}}K_{Tb}^2\right\}^2\right]}$$

Similarly for t = t_{max} to t = t_{sep}, the particular integral is obtained by putting $\frac{\pi}{2(t_{sep}-t_{\max})}=K_{Ta}$

$$\therefore x_i = \frac{\frac{b_{ij}}{c_{ij}}\frac{1}{K_{T}}F_{\max}\cos\{K_{Ta}t_{sep}-K_{Ta}t\}}{c_{ij}\left[-\frac{b_{ij}^2}{c_{ij}^2}K_{Ta}^2-\left\{1-\frac{a_{ij}}{c_{ij}}K_{Ta}^2\right\}^2\right]}+\frac{F_{\max}\left\{1-\frac{a_{ij}}{c_{ij}}K_{Ta}^2\right\}\sin\{K_{Ta}t_{sep}-K_{Ta}t_{\max}\}}{c_{ij}\left[-\frac{b_{ij}^2}{c_{ij}^2}K_{Ta}^2-\left\{1-\frac{a_{ij}}{c_{ij}}K_{Ta}^2\right\}^2\right]}$$

3.4.4 Force as an Exponential Function of Time, $Fc_i(t) = F_{\max i} f(e^t)$

In this case the force is assumed to at an exponential function of time where the force increases exponentially from time t_{hit} to time t_{max} and thereafter it reduces exponentially again from time t_{max} to time t_{sep}.

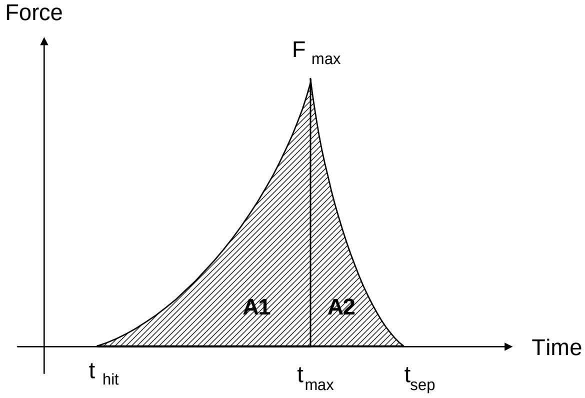


Figure 3.18: Collision force is an exponential function over the contact period.

For	t= 0 to t_{hit}	$F = 0$
	t = t_{hit} to t = t_{max}	$F = F_{\max} e^{t-t_{\max}}$
	t = t_{max} to t = t_{sep}	$F = F_{\max} e^{t_{\max}-t}$
	t= t_{sep} to t = ∝	$F = 0$

The particular integral is obtained as,

For t = t_{hit} to t = t_{max}

$$x_i = \frac{\frac{F_{\max}}{a_{ij}} e^{t-t\max}}{D^2 + \frac{b_{ij}}{a_{ij}} D + \frac{c_{ij}}{a_{ij}}}$$

$$\Rightarrow x_i = \frac{\frac{F_{\max}}{a_{ij}} \left[e^{t-t\max}\right]}{1^2 + \frac{b_{ij}}{a_{ij}} \times 1 + \frac{c_{ij}}{a_{ij}}}$$

$$\therefore x_i = \frac{F_{\max}\left[e^{t-t\max}\right]}{a_{ij} + b_{ij} + c_{ij}}$$

Similarly, for t = t_{max} to t = t_{sep}, it may thus be obtained, $x_i = \frac{F_{\max}\left[e^{t\max-t}\right]}{a_{ij} - b_{ij} + c_{ij}}$

It has been observed form the four different expressions which are derived in the above paragraphs that among the first two there are c_{ij} terms (restoring force coefficient) which exist in multiplied form with the other terms in the denominator. Therefore, if c_{ij} become zero which practically do happen in the case of surge, sway and yaw motions, the amplitudes thus literally become infinity for any particular case. These are practically unacceptable. In this study, therefore, the fourth function is being used. By putting $t = t_{max} = 0$ in the particular integral the maximum amplitude is thus obtained as $x_{i0} = F_{max} / (a_{ij} + b_{ij} + c_{ij})$.

3.5 Validation of the Model

The developed model has been compared with a number of published research works which are described in the following paragraphs.

3.5.1 Comparison of Lost Kinetic Energy

The comparison of loss of kinetic energy has been done using two similar ships of length 116 meter (particulars are given in Table 3.5). The collisions were taken at various angles of attack and speeds as well. The results are compared with published data of Petersen [41], Hanhirova [42] and Zhang [15] as shown in Table 3.6.

Table 3.5 Vessel and collision particulars

Length	116 meters
Breadth	19.0 meters
Draft	6.9 meters
Displacement	10,340 ton
Coefficient of restitution	0.0

It is however, considered in this validation that the hitting takes in place at the centre of the struck ship and the collision is entirely plastic. A plastic collision means that the ships remain in contact after the collision and all the kinetic energy is being used in deforming the ships hull structure and dynamic movement of the ships. The results are being compared with the loss of kinetic energy along 1-axis (KE1) and 2-axis (KE2) directions (defined earlier) and the units expressed here are in mega joule. The comparison suggests that there are noticeable variations among different methods adopted by different researcher; nevertheless, the results do not exceed the comparative limits and thus it may be concluded that the developed model is in good agreement.

Table 3.6: Comparison of loss of kinetic energy

Va (m/s)	Vb (m/s)	α=β (deg.)	KE1 (MJ)				KE2 (MJ)			
			Petersen (1982)	Hanhirova (1995)	Zhang (1999)	Present Study	Petersen (1982)	Hanhirova (1995)	Zhang (1999)	Present Study
4.5	0	90	0	0	0	0	69.6	54.4	70.1	78.519
4.5	4.5	90	24.7	41.5	21.4	26.173	64.1	54.4	70.1	78.519
4.5	4.5	60	5.2	15.8	0.2	32.716	28.8	28.3	35.3	58.889
4.5	4.5	30	49.3	7.2	0	12.616	71.9	4	7.4	19.629
4.5	0	120	9.8	14	15	19.629	54	40.9	50.1	58.889
4.5	2.25	120	40.7	51.5	45.1	26.173	60.3	42.8	57.5	58.889

3.5.2 The Hydrodynamic Coefficients

The hydrodynamic coefficients a_{ij}, b_{ij} and c_{ij} depend on the hull form and the interaction between the hull and surrounding water. The coefficients may also vary during a collision as well and the range of variation is even wider considering open or restricted water conditions. However, for simplicity Minorsky (1959) proposed to use a constant value of the added mass coefficients of ships for the sway motion, $m_{ay} = a_{22} = 0.4$.

Motora et.al [43] conducted a series of tests in determining the added mass coefficients for sway motion and has found that the coefficient varies in the range of 0.2 to 1.3. The study revealed that the longer the duration, the larger the value of the coefficient. The added mass coefficient related to forward motion is found to be relatively smaller and in the range of, $m_{ax} = a_{11}$ = 0.02 to 0.07.

The added mass coefficient for rolling is suggested by Bhattacharyya (1978) to be in between 10 to 20 percent of the actual displacement of the ship. The added mass coefficient for yaw motion of the ship, j_a, is used by Pedersen et. al [44] as 0.21. Crake (1998) in his research work used an empirical relation for finding the added mass in yaw as $a_{66} = 0.0991\rho T^2 L_{BP}^3$.

However, in this study the hydrodynamic coefficients were determined using the 3-D source distribution method [45] and the values are compared with existing results expressed in range of virtual mass in Table 3.4.

Table 3.4 Comparison of virtual mass.

Hydrodynamic Coefficients	Range of Virtual Mass (non dimensional)	46 m Vessel (3-D Method)	32 m Vessel (3-D Method)
Surge, a_{11}	1.02 – 1.07	1.01	1.01
Sway, a_{22}	1.20 – 2.30	1.05	1.14
Roll, a_{44}	1.10 – 1.20	1.61	1.22
Yaw, a_{66}	1.20 – 1.75	1.40	1.53

It is observed from the comparison that the hydrodynamic coefficients for surge, sway and yaw fairly matches within the range except a few discrepancies in the sway motion. This is probably because the range is determined on the basis of ships that are relatively large and ocean going in comparison to the small vessels designed for inland transportation in Bangladesh.

Chapter 4

RESULTS AND DISCUSSIONS

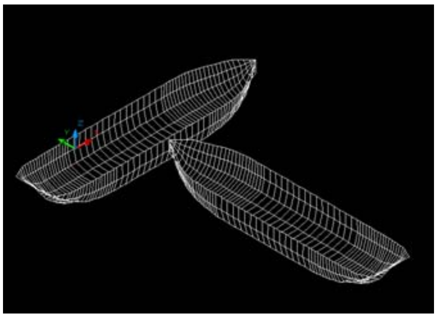
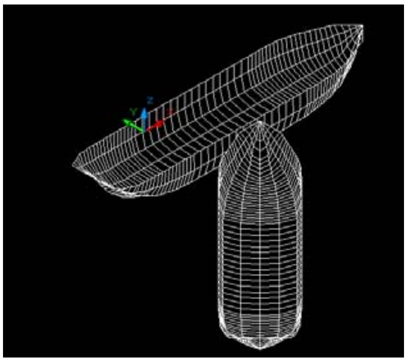
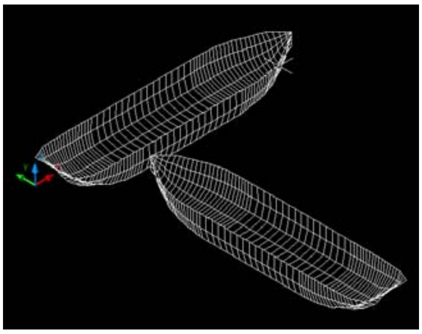
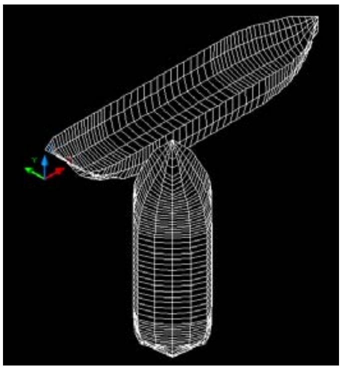
4.1 General

This chapter reveals the results obtained from the numerical investigations and discusses the various facts related to the analyses. In the present study two different vessels (principle particulars are given in Table 4.1) were considered for conducting numerical investigation in 22 different scenarios which are broadly categorised in four different groups, as shown in Table 4.2. Both the vessels are common inland vessels and such hull shapes are generally been used both in cargo and passenger ships. Results are presented in two different categories, (1) Analysis of non dimensional forces due to changes in different variables and (2) Time domain simulation due to changes in different variables. The variables considered in the study are the speed of the striking ship, collision time, added mass for sway direction, location of hitting and collision angle. The chapter also discusses the use of Collision Avoidance Chart (CAC) with examples.

Table 4.1 Principle particulars of the ships

Principle Particulars		
Ship 1 (46 Meter)	Length	46.800 meter
	Breadth	10.564 meter
	Draft	2.3340 Meter
	Displacement	556.3 Tone
	Angle of vanishing stability	67 degree
Ship 2 (32 Meter)	Length	30.640 meter
	Breadth	6.700 meter
	Draft	3.500 Meter
	Displacement	498.0 Tone
	Angle of vanishing stability	75 degree

Table 4.2: Four principle categories of collision scenarios.

	Ship striking at 90 degree angle of attack at amidship
	Ship striking at 45 degree angle of attack at amidship
	Ship striking at 90 degree angle of attack at L/4 aft of amidship
	Ship striking at 45 degree angle of attack at L/4 aft of amidship

4.2 Analysis on Forces

It is revealed in Figure 4.1 that the force in sway direction varied in a range up to 6 times of the displacement or buoyancy force due to variation in striking ship speed. The speed is taken up to 12 knots (6.1782 m/sec) since generally this is the maximum allowable speed in the inland waterways. Graphically, the higher the speed of the striking ship, the higher the force in sway direction. Thus, the force in sway direction is proportional with the striking ship speed. It is also observed that at higher speeds the coefficient of restitution with lower value plays a very important role by reducing the collision force significantly.

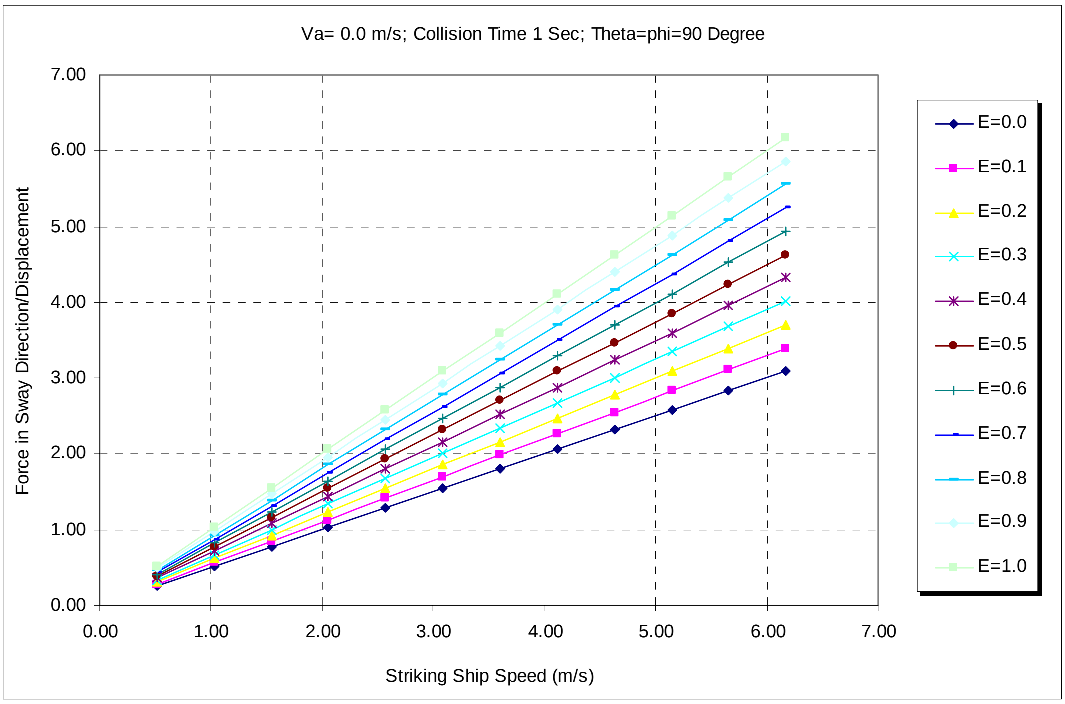


Figure 4.1: Force in sway direction vs. striking speed at various restitutions.

Figure 4.2 suggests that the collision force substantially decreases with the increase in collision/contact period. For a very short collision period the variations of force in the sway direction due to different coefficients of restitution are very large but for a shorter collision period the variations are comparatively small and practically trivial.

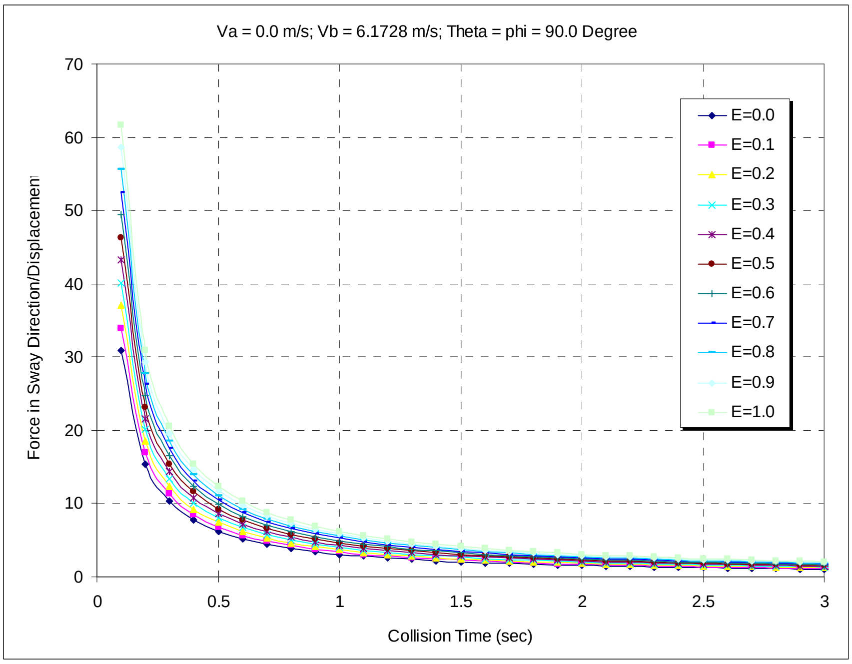


Figure 4.2: Force in sway direction vs. collision time at various restitutions.

The collision force is also notably affected by the added mass of the ship. It is observed from Figure 4.3 that higher the added mass the lower is the collision force and vice versa. It is observed from the figure that for lower added mass the force varies relatively largely with the variation in coefficient of restitution while the force varies relatively less at higher added mass.

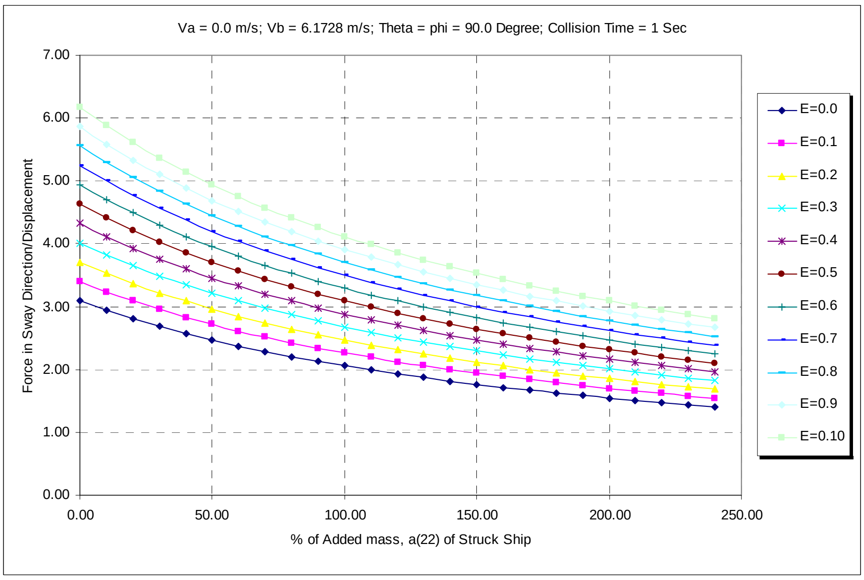


Figure 4.3: Force in sway direction vs. added mass for various restitutions.

The variation in collision angle also significantly affects the collision force. Figure 4.4 and 4.5 shows the non-dimensional force in surge and sway direction respectively for various coefficients of restitutions and angle of attack. In this particular graph the striking ship speed is assumed to be 12 knots and the speed of the struck ship is assumed to be 0 (zero) knot. The collision duration is taken to be 1 second.

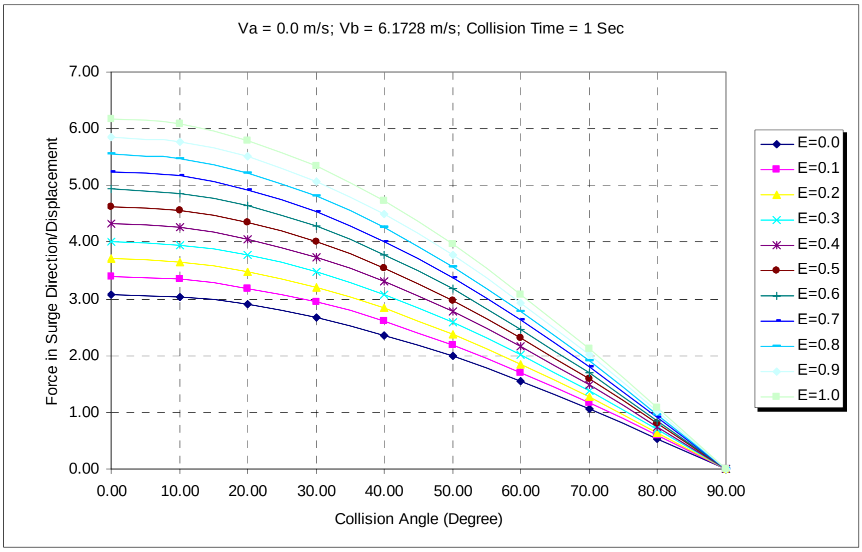


Figure 4.4: Force in surge direction for various collision angle & restitution.

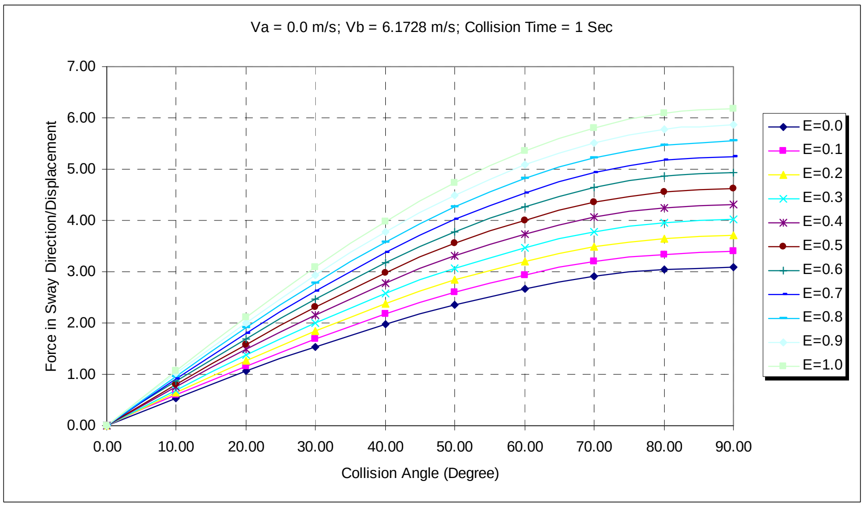


Figure 4.5: Force in sway direction for various collision angle & restitution.

4.3 Time Domain Simulation for Different Cases

Time domain simulations of different cases were performed and represented in Figure 4.6 to Figure 4.92. In the first eight cases (described in section 4.3.1 to section 4.3.8) collision between two similar vessels, e.g. a 46 meter vessel strikes another 46 m vessel, are studied for different angle of attack, speed and point of hitting. In sections 4.3.9 to 4.3.16 the collision between 32 meter vessels and another same 32 meter vessel is considered. Finally, in section 4.3.17 to 4.3.22 the collision between a 32 meter vessel and another 46 meter vessel is considered where the 46 meter vessel strikes the 32 meter vessel at different speeds and locations. A summary of these scenarios are given in Table 4.3.

It is observed from Figures 4.6, 4.9, 4.13, 4.17, 4.22, 4.26, 4.30 and 4.35 that for 90 degree collision angle the surge motion is very much negligible and the motion very significantly increases as the collision angle changes from 90 degree to 45 degree. It is indeed notable that the differences in motion amplitude are very distinct for various coefficients of restitutions in all the cases irrespective of collision angle, suggesting that this particular parameter can significantly reduce surge motion in lower restitution. This findings, however, been re-established by applying different collision scenarios, e.g. collision between 32 m vessel and 32 m vessel and collision between 32 meter vessel and 46 meter vessel, as shown in sections 4.3.9 to 4.3.22.

The Figures for surge motions (4.40, 4.43, 4.46, 4.51, 4.55, 4.58, 4.61, 4.66, 4.71, 4.74, 4.76, 4.80, 4.85 and 4.90) suggest that the surge amplitudes are indeed distinctly separable for various coefficients of restitutions and thus motions can be reduced significantly (almost 83%) by using the right kind of material at the outer envelop of the ships.

Table 4.3: Summary of the collision scenarios investigated in this study.

Case No.	Name of the Struck Ship	Name of the Striking Ship	Speed of the Striking Ship (knots)	Location of hitting along the length	Angle of attack (Degrees)
1	46 Meter Vessel	46 Meter Vessel	1.0	Midship	90.0
2	46 Meter Vessel	46 Meter Vessel	1.0	Midship	45.0
3	46 Meter Vessel	46 Meter Vessel	1.0	L/4 aft of Midship	90.0
4	46 Meter Vessel	46 Meter Vessel	1.0	L/4 aft of Midship	45.0
5	46 Meter Vessel	46 Meter Vessel	6.0	Midship	90.0
6	46 Meter Vessel	46 Meter Vessel	6.0	Midship	45.0
7	46 Meter Vessel	46 Meter Vessel	6.0	L/4 aft of Midship	90.0
8	46 Meter Vessel	46 Meter Vessel	6.0	L/4 aft of Midship	45.0
9	32 Meter Vessel	32 Meter Vessel	1.0	Midship	90.0
10	32 Meter Vessel	32 Meter Vessel	1.0	Midship	45.0
11	32 Meter Vessel	32 Meter Vessel	1.0	L/4 aft of Midship	90.0
12	32 Meter Vessel	32 Meter Vessel	1.0	L/4 aft of Midship	45.0
13	32 Meter Vessel	32 Meter Vessel	6.0	Midship	90.0
14	32 Meter Vessel	32 Meter Vessel	6.0	Midship	45.0
15	32 Meter Vessel	32 Meter Vessel	6.0	L/4 aft of Midship	90.0
16	32 Meter Vessel	32 Meter Vessel	6.0	L/4 aft of Midship	45.0
17	32 Meter Vessel	46 Meter Vessel	1.0	Midship	90.0
18	32 Meter Vessel	46 Meter Vessel	3.0	Midship	90.0
19	32 Meter Vessel	46 Meter Vessel	6.0	Midship	90.0
20	32 Meter Vessel	46 Meter Vessel	1.0	L/4 aft of Midship	90.0
21	32 Meter Vessel	46 Meter Vessel	3.0	L/4 aft of Midship	90.0
22	32 Meter Vessel	46 Meter Vessel	6.0	L/4 aft of Midship	90.0

Investigation of sway motions suggest that the motion amplitudes are maximum at 90 degree collision angle of attack and it reduces as the angle changes from 90 degree to 45 degree. The motion amplitude has been found to be proportional to the striking ship's speed and coefficient of restitution, as shown in Figures 4.7, 4.10, 4.14, 4.18, 4.23, 4.27, 4.31, 4.36, 4.41, 4.44, 4.47, 4.52, 4.56, 4.59, 4.62, 4.66, 4.72, 4.75, 4.78, 4.81, 4.86 and 4.91. It is further observed that the motion can also be significantly reduced by up to 85 percent if zero restitution materials are being used at the ship surface. Indeed this application appears to be effective for preventing unwanted dynamic movements of ships during high speed collisions.

The most important aspect of this study is to investigate the roll characteristics for different collision scenarios and this is been performed for collision between both similar vessels and dissimilar vessels at different speeds, collision and angle and location of hitting. Some interesting patterns of roll amplitudes have been revealed in Figures 4.8, 4.11, 4.15, 4.19, 4.24, 4.28, 4.32, 4.37, 4.42, 4.45, 4.48, 4.53, 4.57, 4.60, 4.63, 4.68, 4.73, 4.76, 4.79, 4.82, 4.87 and 4.92.

Depending up on the ships very own particular geometric characteristics, which include the hydrodynamic coefficients, the roll motion in time domain shows very unique patterns. It is clearly visible that for collision between 46 m and 46 m vessel, the roll motion of the struck vessel is of very high frequency in comparison to the collision responses of 32 m vessel. This clearly suggests that a collision on 46 meter vessel will surely cause discomforting movement and provide damage to the ship and injury to the passengers while the 32m vessel appears to be rolling at lower frequency and damping out the blow received from the striking ship slowly with time.

The most important of the findings are the roll amplitude against different coefficient of restitutions. It is observed that higher striking speed causes higher the moment for rolling and thus higher rolling amplitude. But this amplitude can be reduced significantly up to 83 percent if zero restitution materials are being used. This is indeed, very important aspect of the study that as excessive rolling causes ships to capsize and such capsizing could be prevented by applying the lower restitution shock absorbing materials. This phenomena is simulated in Figure 4.57, 4.60, 4.63, 4.68, 4.79 and 4.92 where the ships roll over the angle of vanishing stability if the coefficient of restitutions are 1 for all the cases. These roll amplitudes, however, are significantly less in their respective cases if the coefficient of restitutions were considered zero or close to zero. Therefore, the facts revealed here could be a mater of life and death and indeed requires due importance to be looked into while construction ship fenders and other similar protective devices.

The pitch and yaw motions have also been investigated along with others motions as well. The studies reveal that the pitch amplitudes are not as significant as other motion amplitudes due to the fact that the moment causing ship to pitch is significantly smaller as the lever of the moment is comparatively small.

The study on yaw motion is considered when ships are being struck at L/4 meter aft of amidship. In most of the cases the yaw motion damps out significantly faster. The variation due to restitution is also very prominently visible and it is observed to reduce around 75 percent of the motion by using the fully plastic material (E=0.0).

4.3.1 Case 1: 46m vs. 46m hits amidship at 90º at speed 1 knot

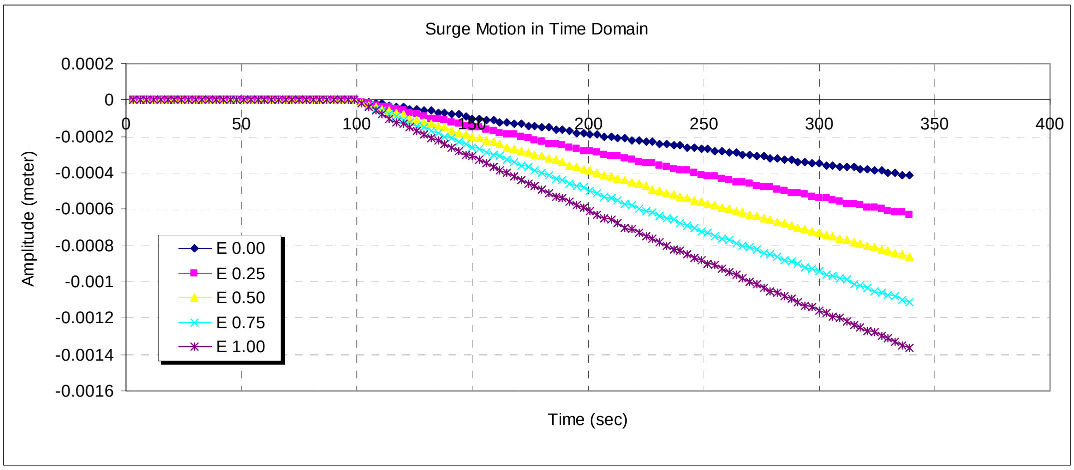


Figure 4.6: Surge motion for various restitutions (struck at 1 knot).

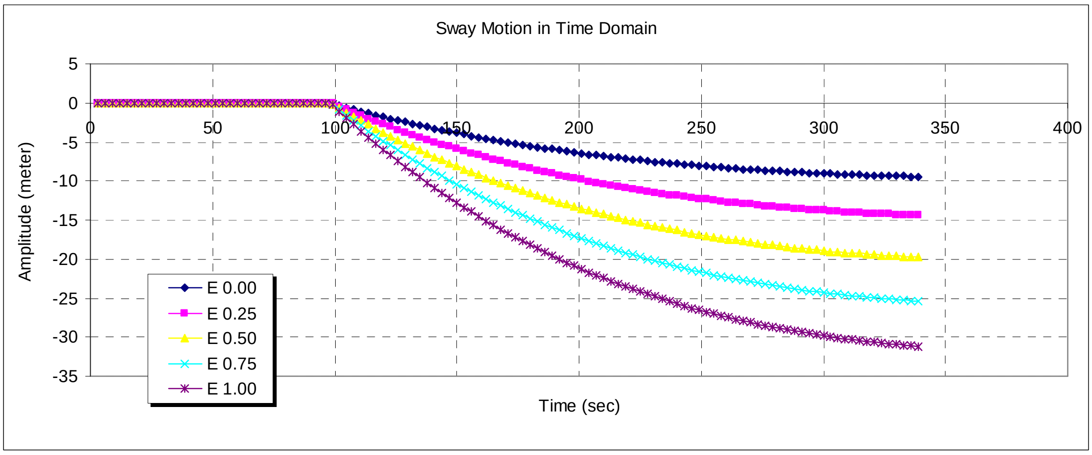


Figure 4.7: Sway motion for various restitutions (struck at 1 knot).

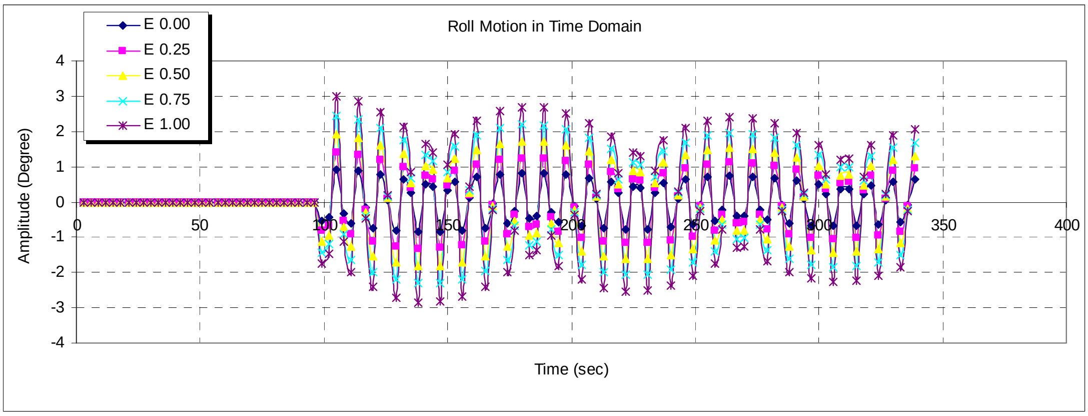


Figure 4.8: Roll motion for various restitutions (struck at 1 knot).

4.3.2. Case 2: 46m vs. 46m hits amidship at 45º at speed 1 knot

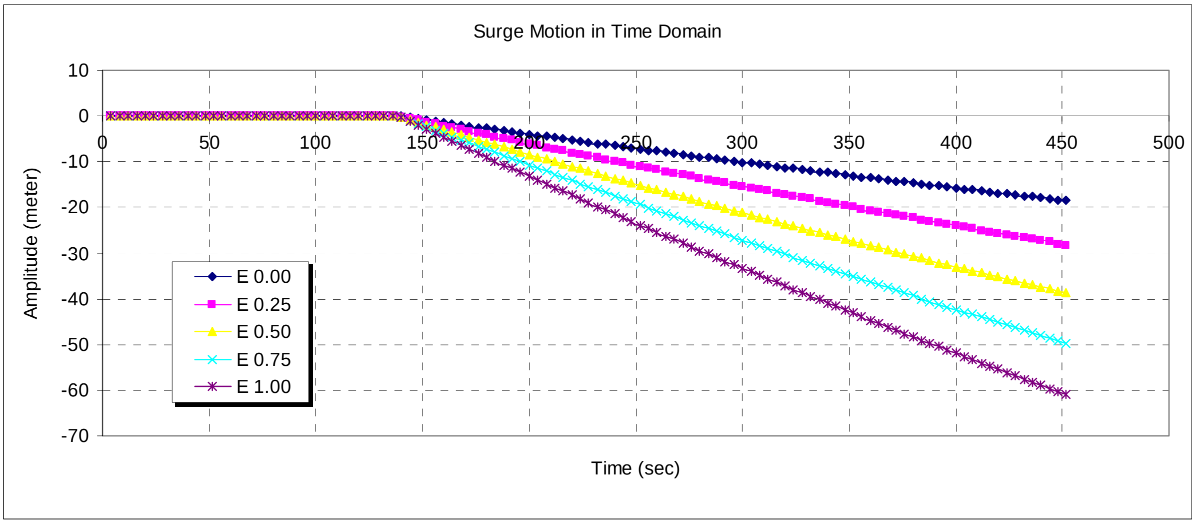


Figure 4.9: Surge motion for various restitutions (struck at 1 knot).

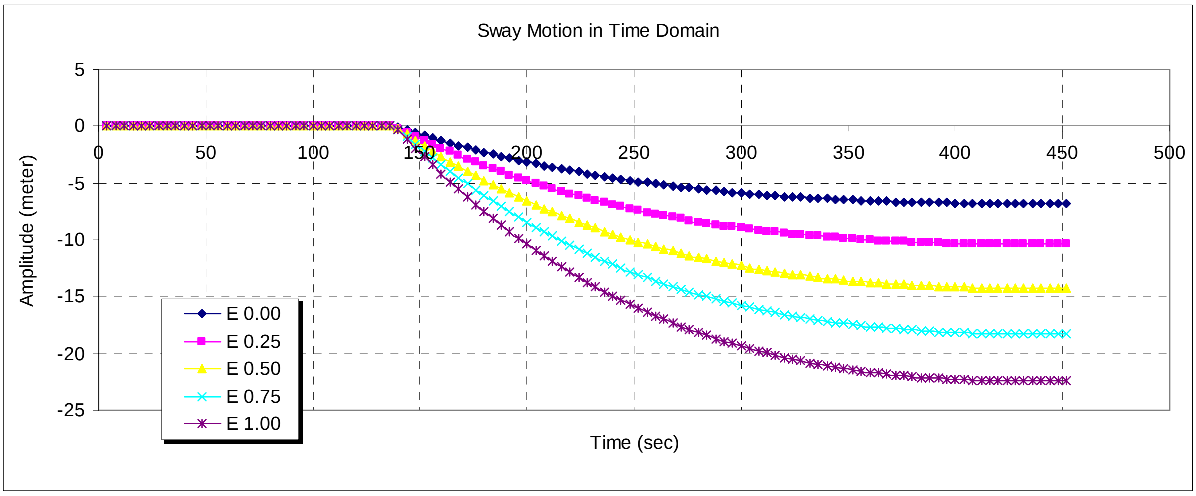


Figure 4.10: Sway motion for various restitutions (struck at 1 knot).

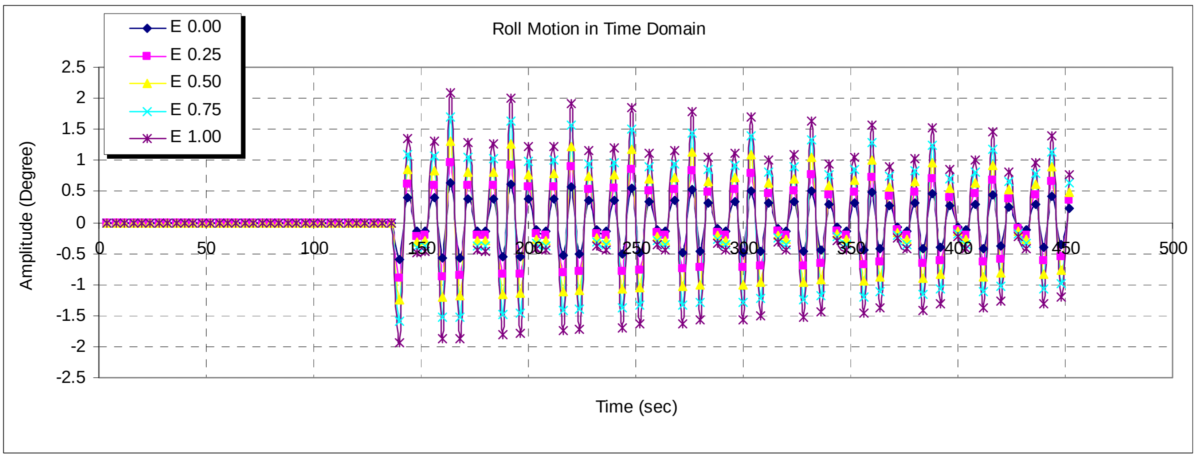


Figure 4.11: Roll motion for various restitutions (struck at 1 knot).

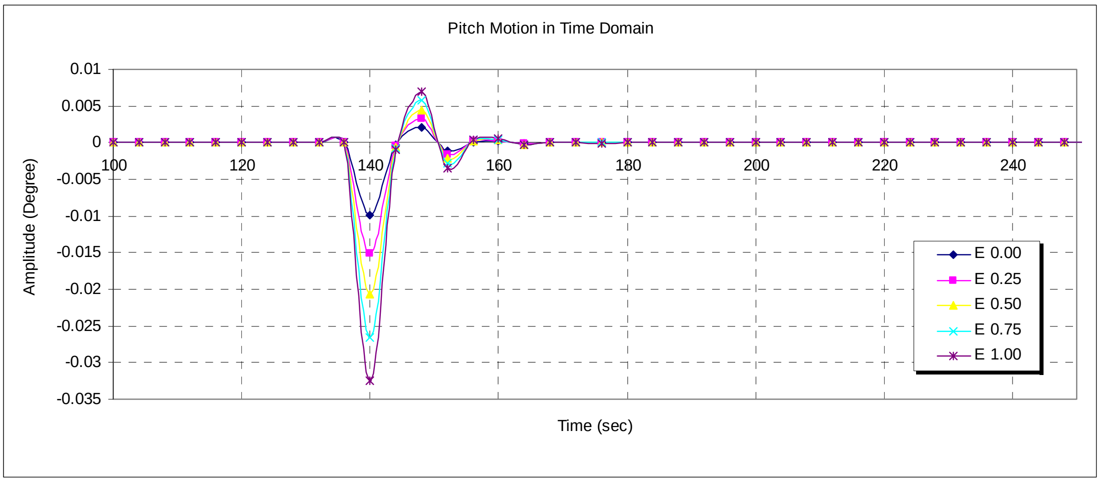


Figure 4.12: Pitch motion for various restitutions (struck at 1 knot).

4.3.3 Case 3: 46m vs. 46m at L/4 aft midship at 90º at speed 1 knot

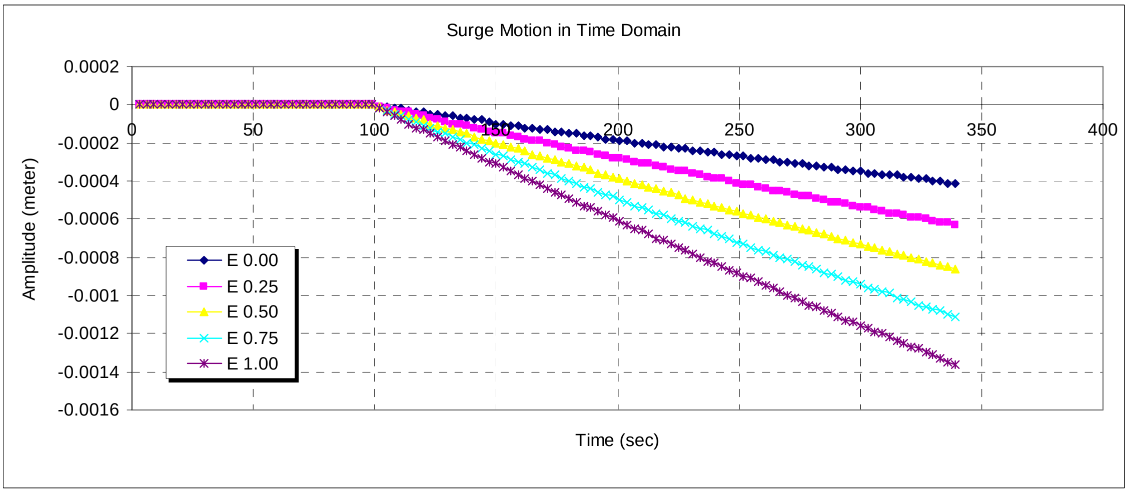


Figure 4.13: Surge motion for various restitutions (struck at 1 knot).

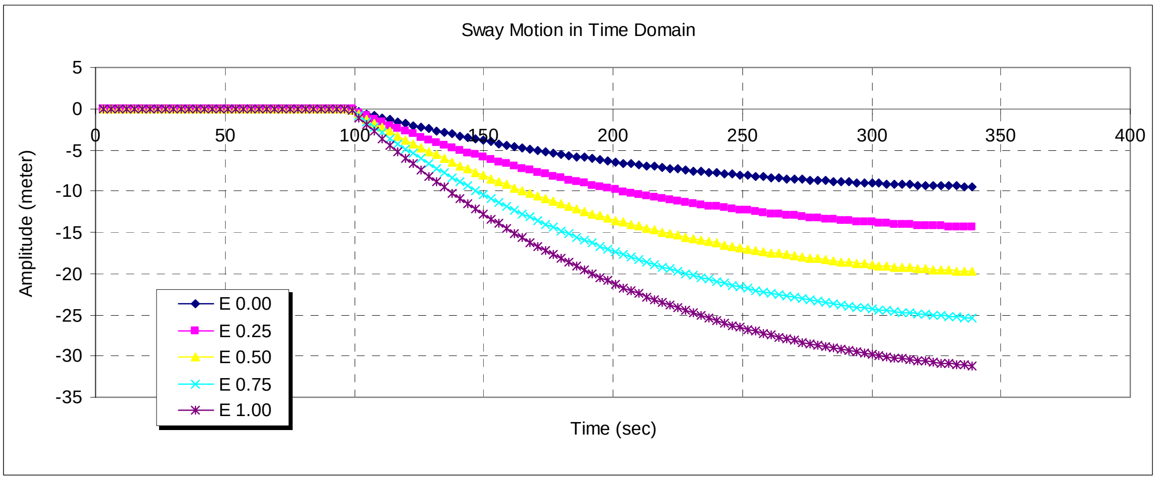


Figure 4.14: Sway motion for various restitutions (struck at 1 knot).

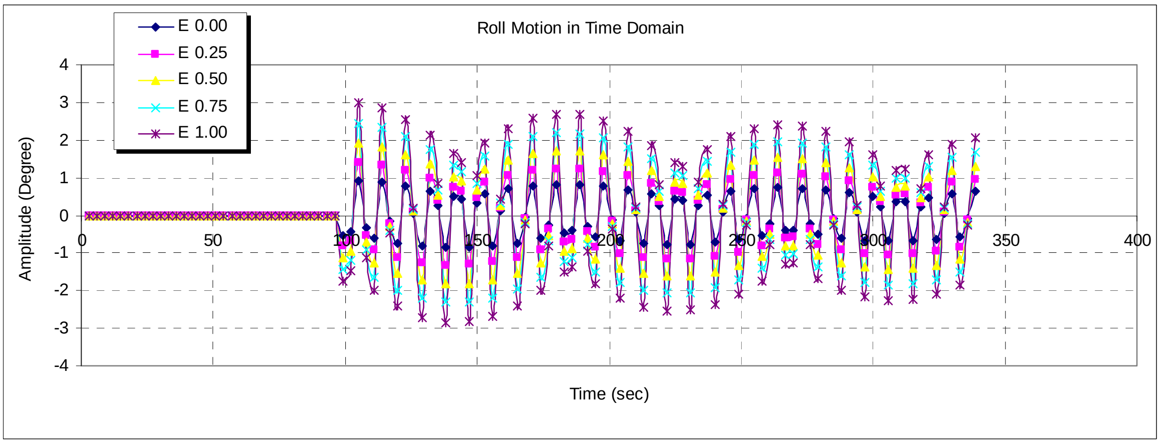


Figure 4.15: Roll motion for various restitutions (struck at 1 knot).

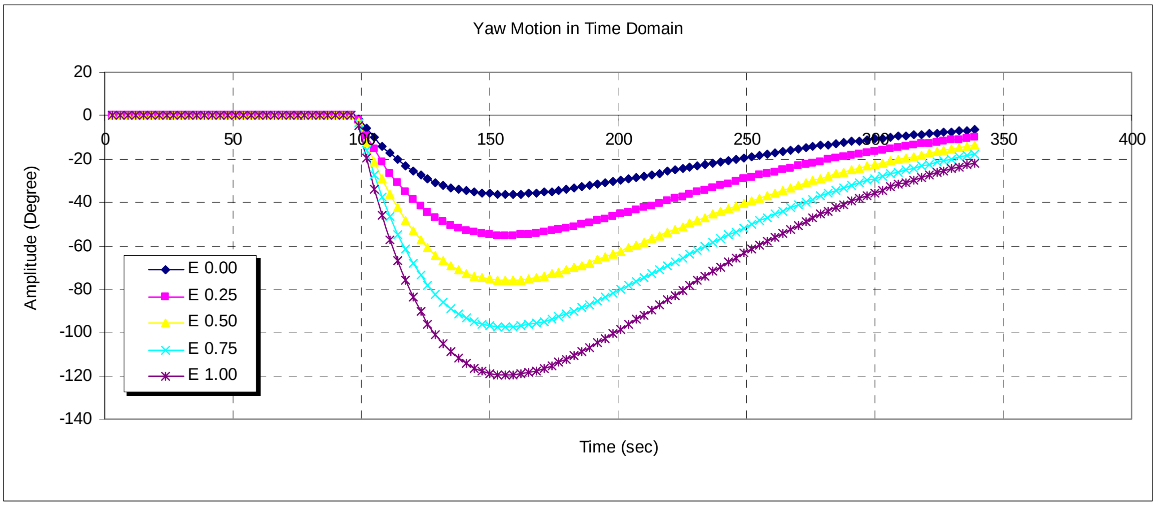


Figure 4.16: Yaw motion for various restitutions (struck at 1 knot).

4.3.4 Case 4: 46m vs. 46m at L/4 aft midship at 45 ° at speed 1 knot

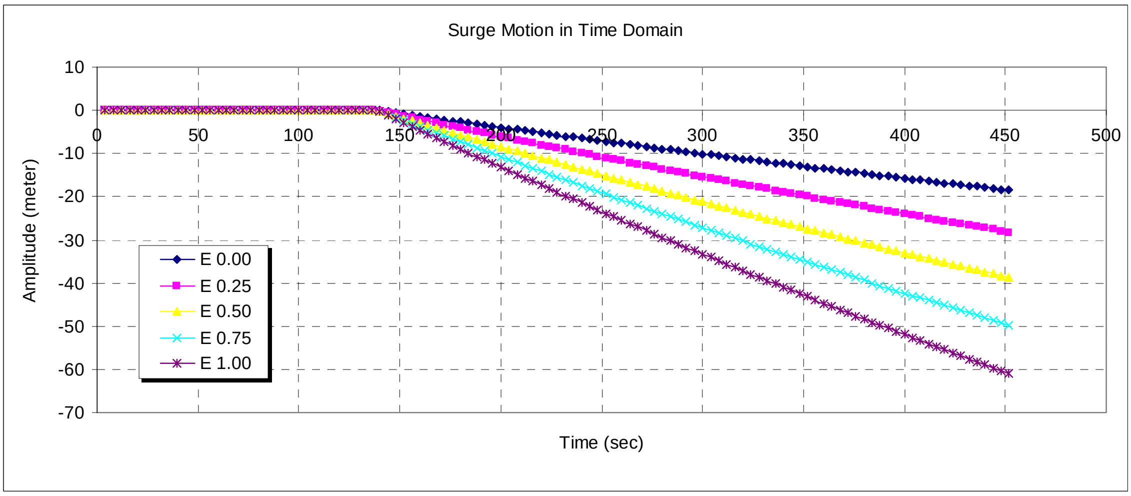


Figure 4.17: Surge motion for various restitutions (struck at 1 knot).

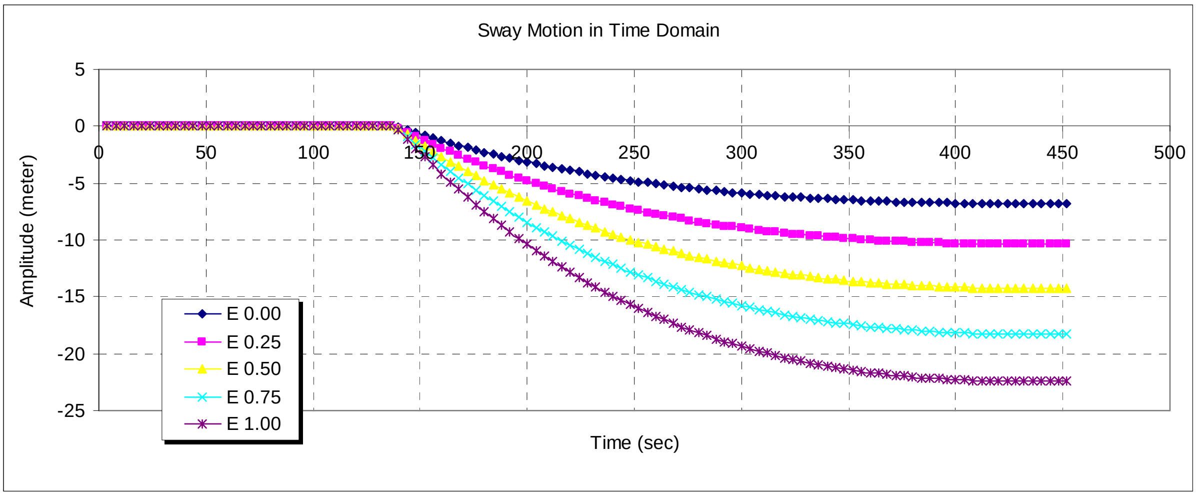


Figure 4.18: Sway motion for various restitutions (struck at 1 knot).

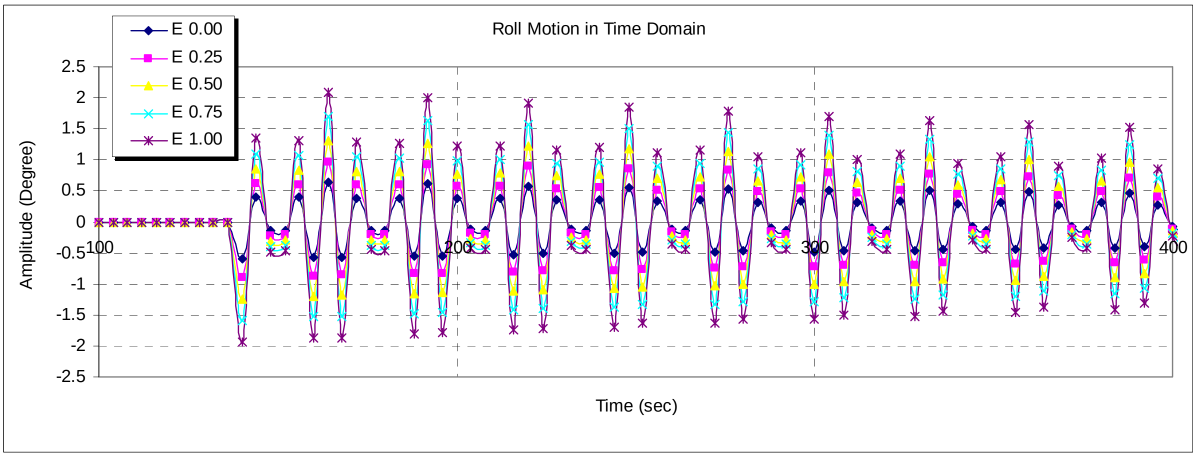


Figure 4.19: Roll motion for various restitutions (struck at 1 knot).

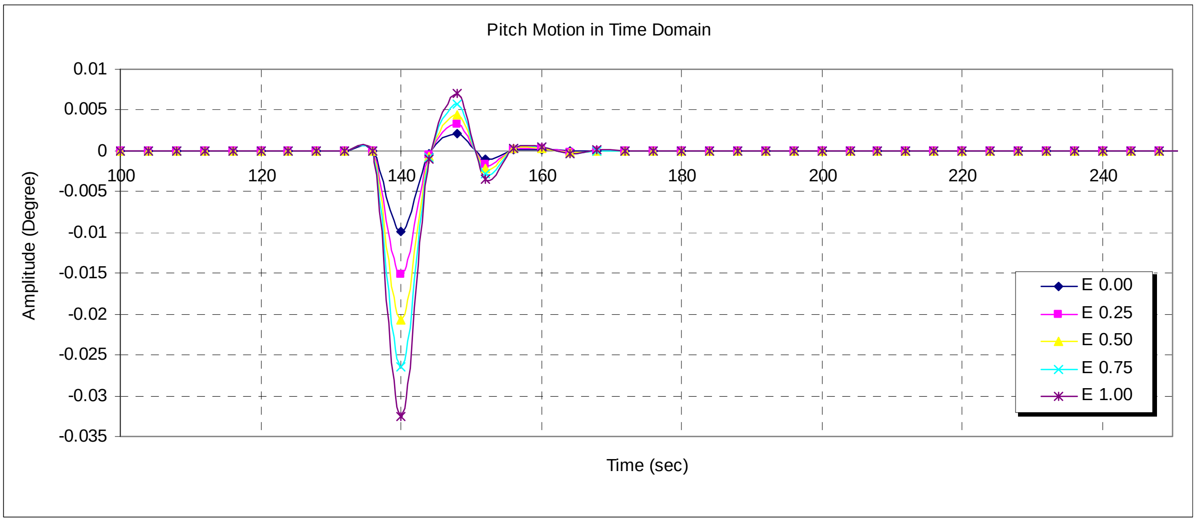


Figure 4.20: Pitch motion for various restitutions (struck at 1 knot).

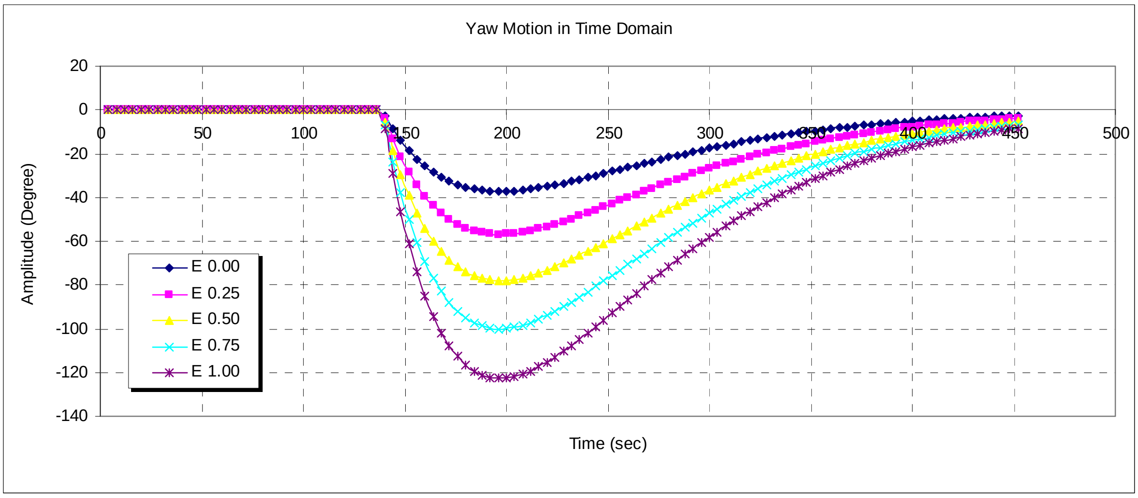


Figure 4.21: Yaw motion for various restitutions (struck at 1 knot).

4.3.5 Case 5: 46m vs. 46m hits amidship at 90º at speed 6 knot

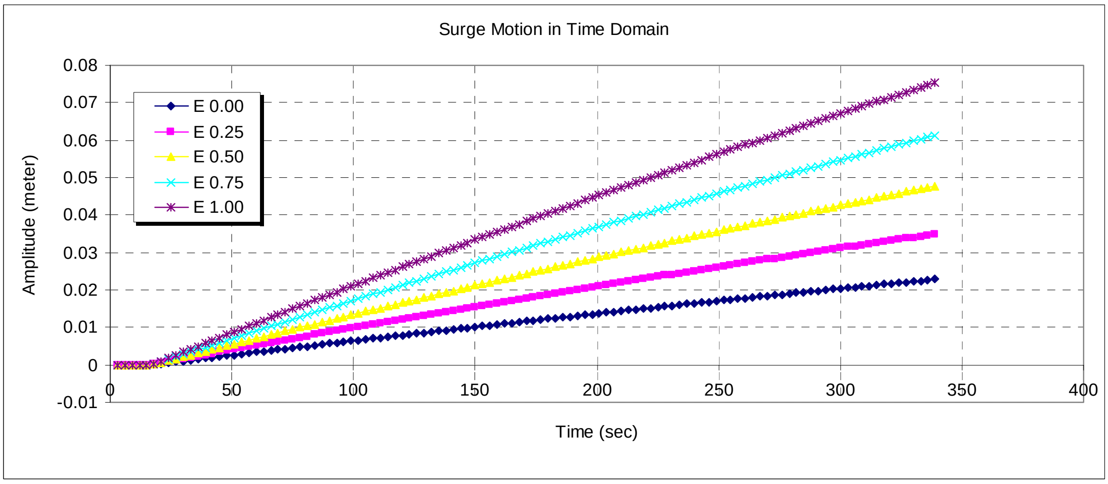


Figure 4.22: Surge motion for various restitutions (struck at 6 knot).

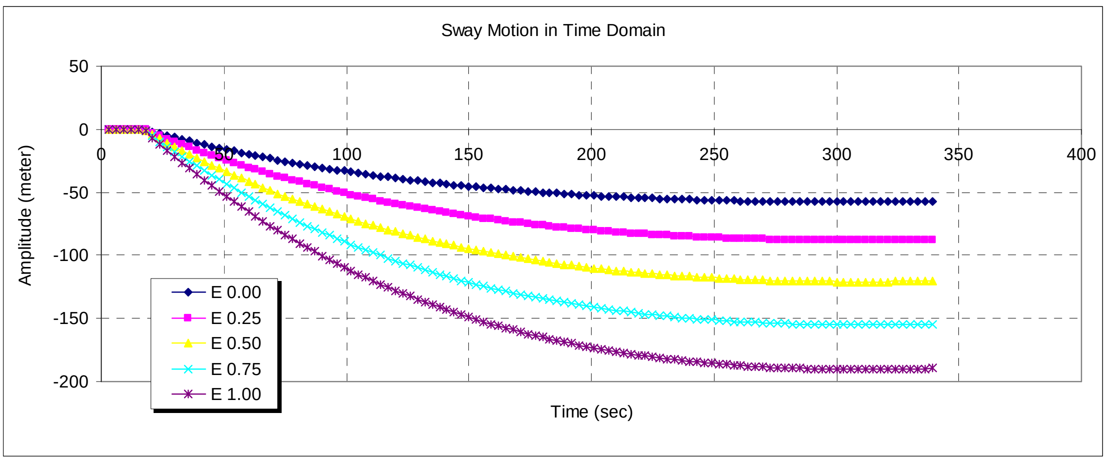


Figure 4.23: Sway motion for various restitutions (struck at 6 knot).

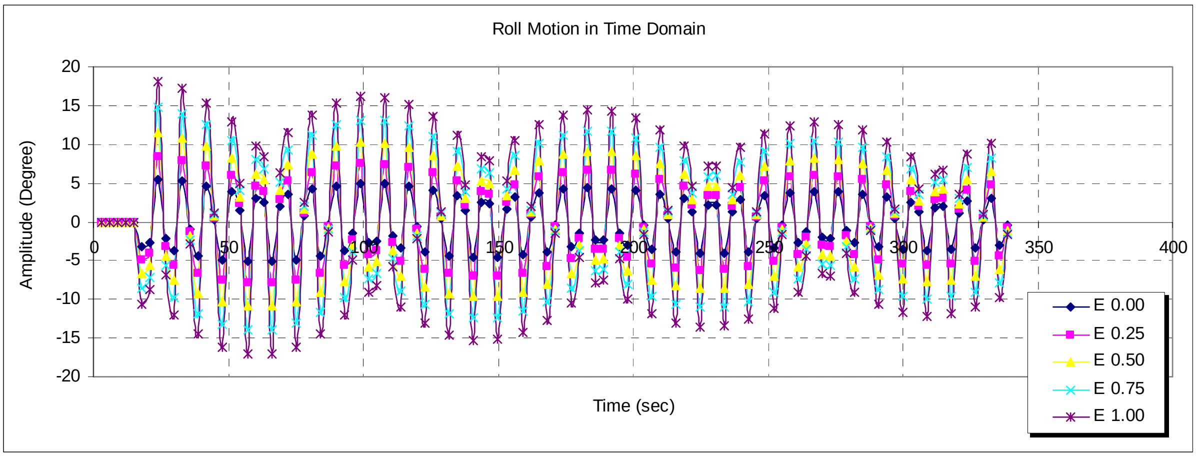


Figure 4.24: Roll motion for various restitutions (struck at 6 knot).

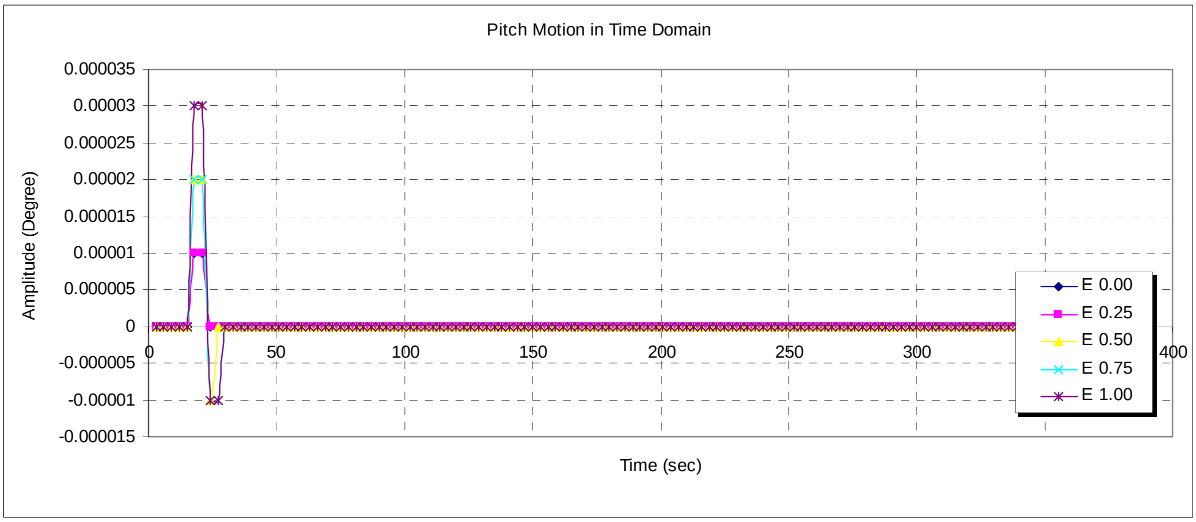


Figure 4.25: Pitch motion for various restitutions (struck at 6 knot).

4.3.6 Case 6: 46m vs. 46m hits amidship at 45° at speed 6 knot

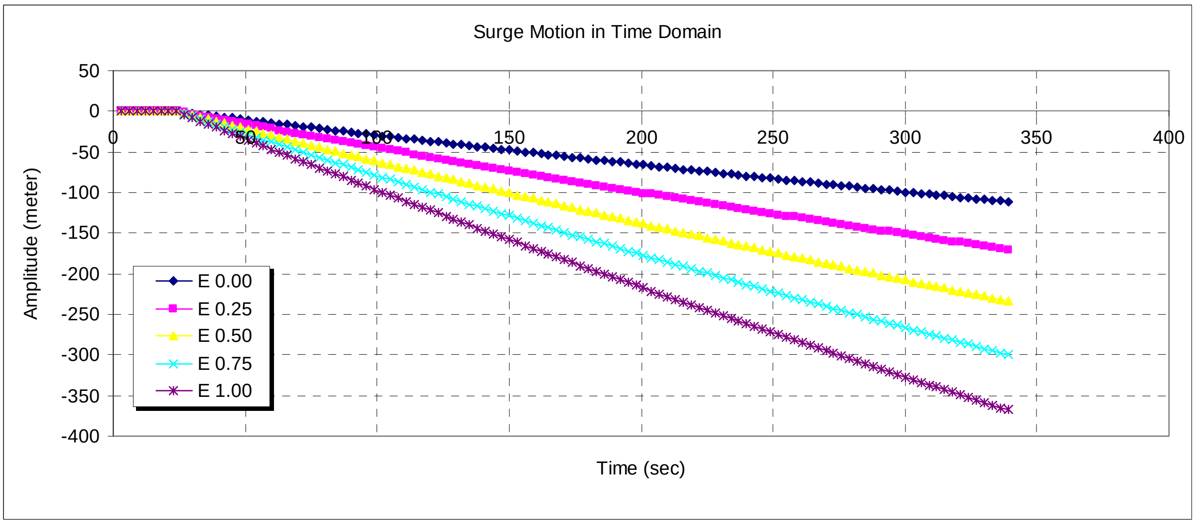


Figure 4.26: Surge motion for various restitutions (struck at 6 knot).

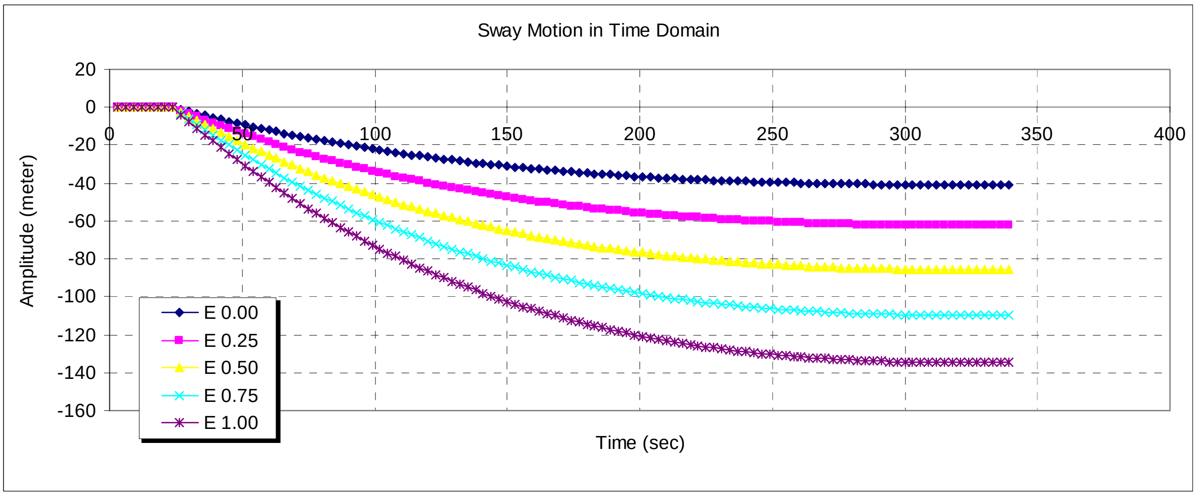


Figure 4.27: Sway motion for various restitutions (struck at 6 knot).

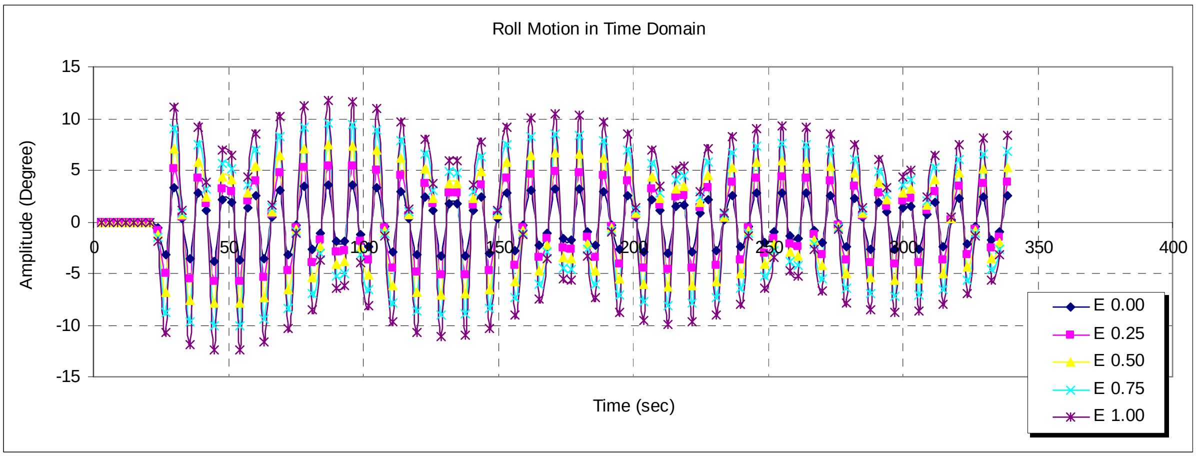


Figure 4.28: Roll motion for various restitutions (struck at 6 knot).

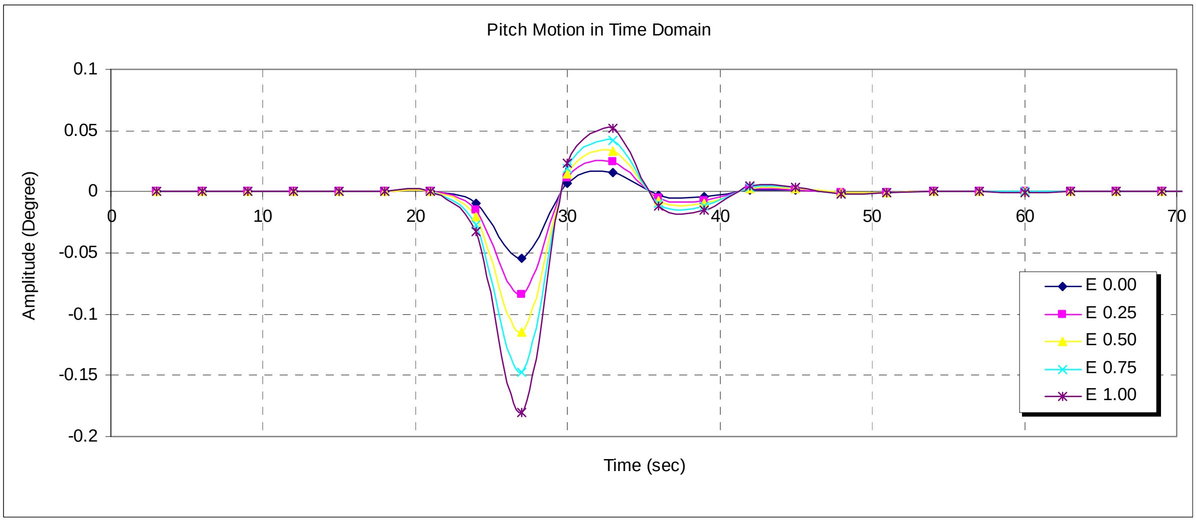


Figure 4.29: Pitch motion for various restitutions (struck at 6 knot).

4.3.7 Case 7: 46m vs. 46m at L/4 aft midship at 90° at speed 6 knot

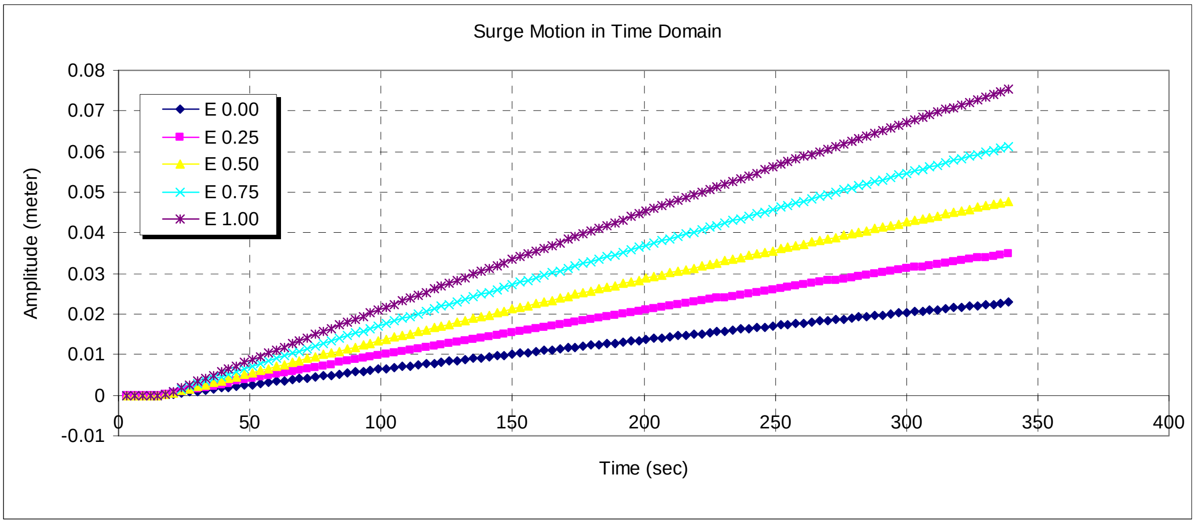


Figure 4.30: Surge motion for various restitutions (struck at 6 knot).

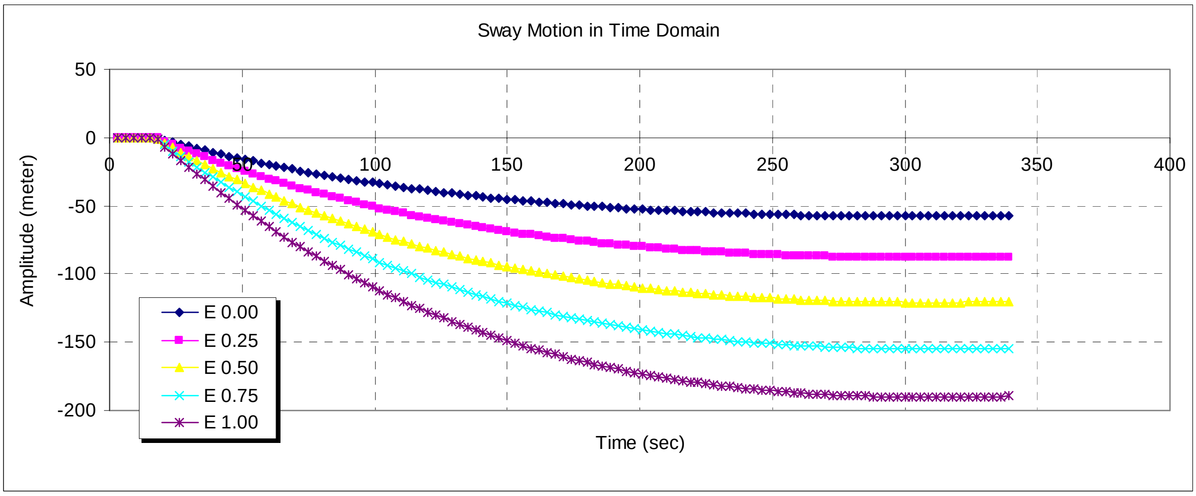


Figure 4.31: Sway motion for various restitutions (struck at 6 knot).

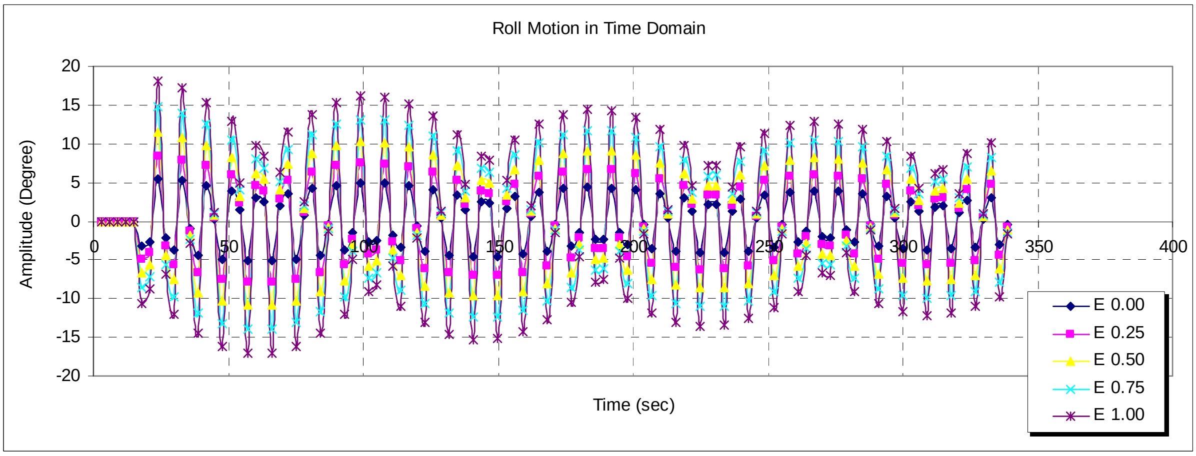


Figure 4.32: Roll motion for various restitutions (struck at 6 knot).

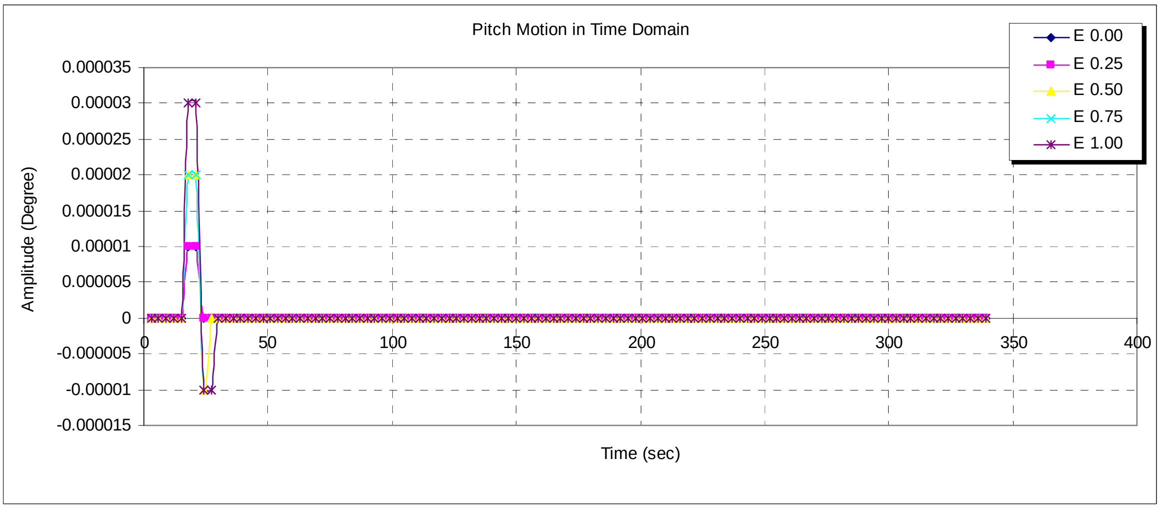


Figure 4.33: Pitch motion for various restitutions (struck at 6 knot).

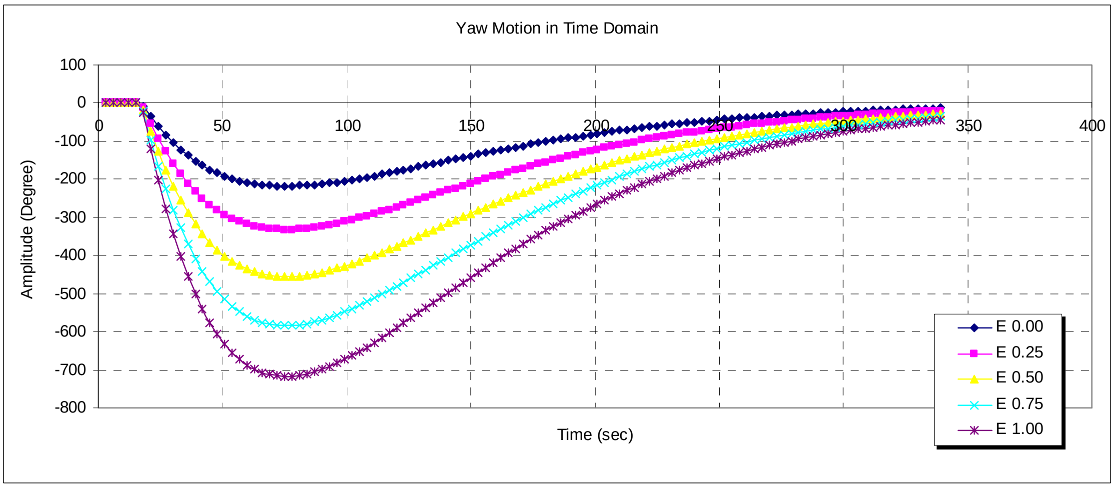


Figure 4.34: Yaw motion for various restitutions (struck at 6 knot).

4.3.8 Case 8: 46m vs. 46m at L/4 aft midship at 45 ° at speed 6 knot

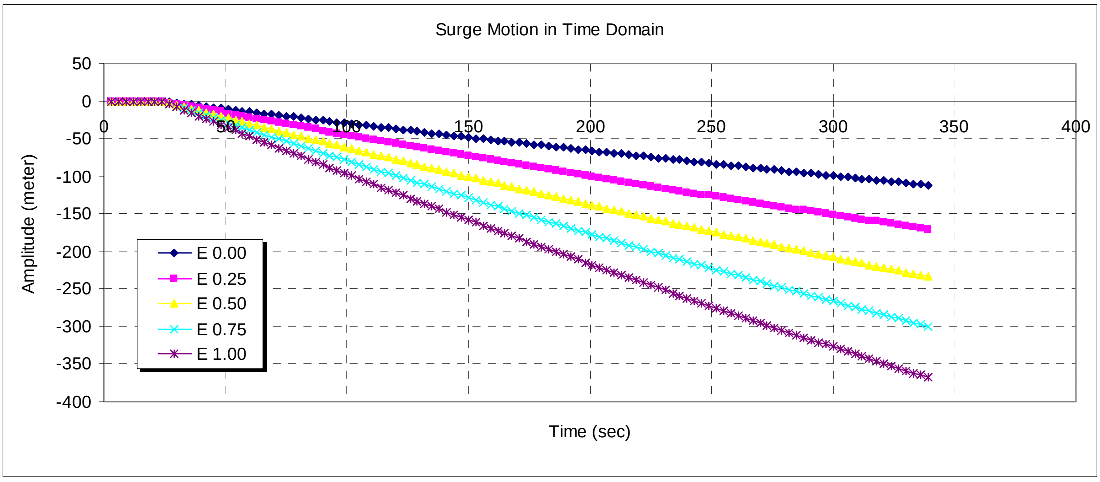


Figure 4.35: Surge motion for various restitutions (struck at 6 knot).

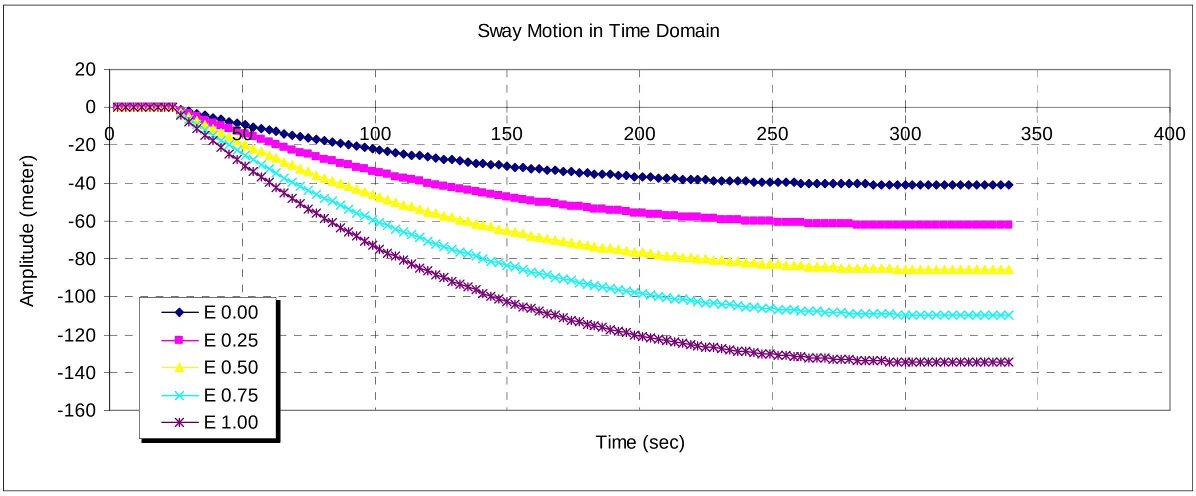


Figure 4.36: Sway motion for various restitutions (struck at 6 knot).

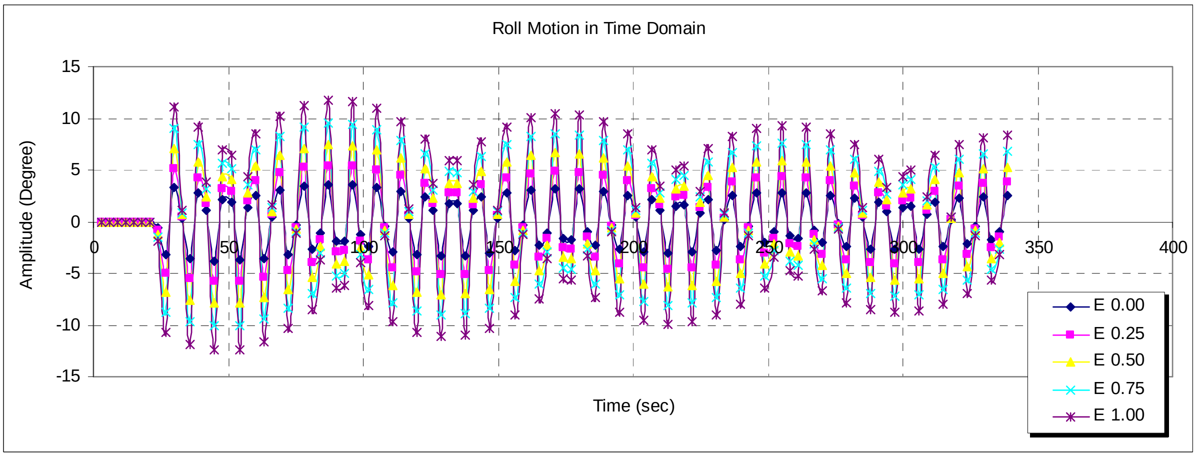


Figure 4.37: Roll motion for various restitutions (struck at 6 knot).

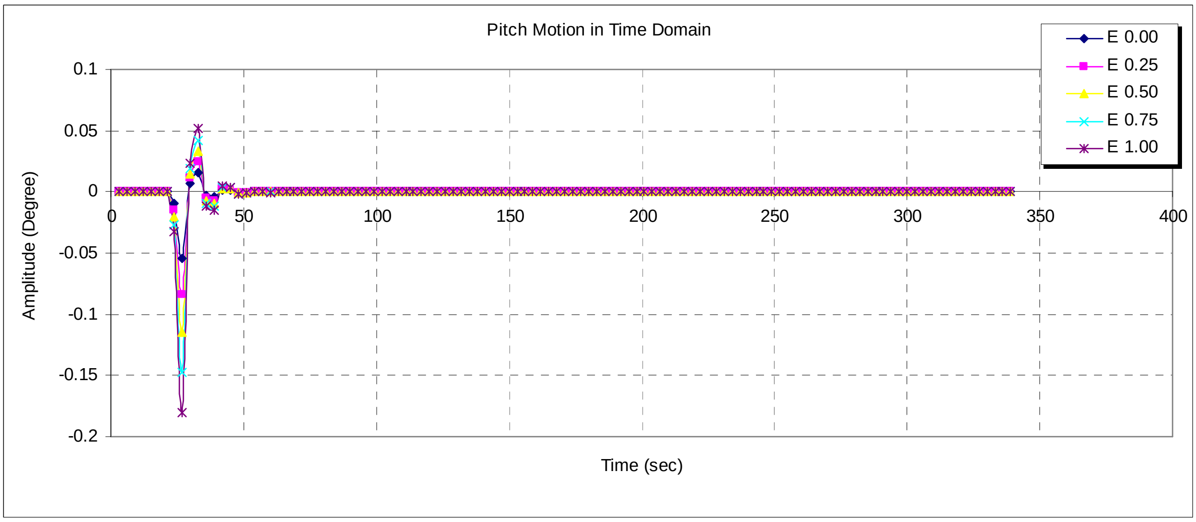


Figure 4.38: Pitch motion for various restitutions (struck at 6 knot).

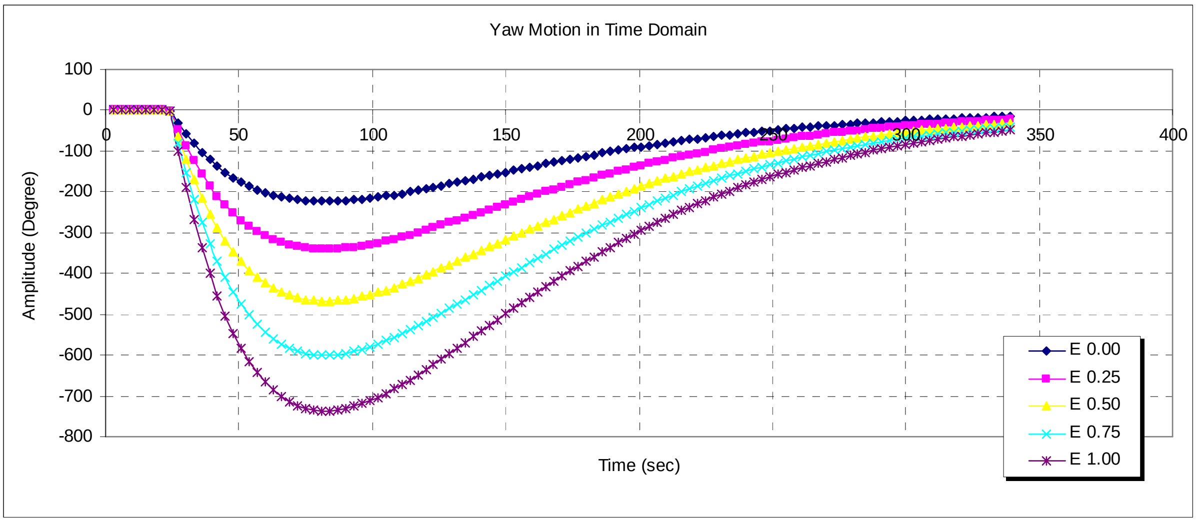


Figure 4.39: Yaw motion for various restitutions (struck at 6 knot).

4.3.9 Case 9: 32m vs. 32m hits amidship at 90° at speed 1 knot

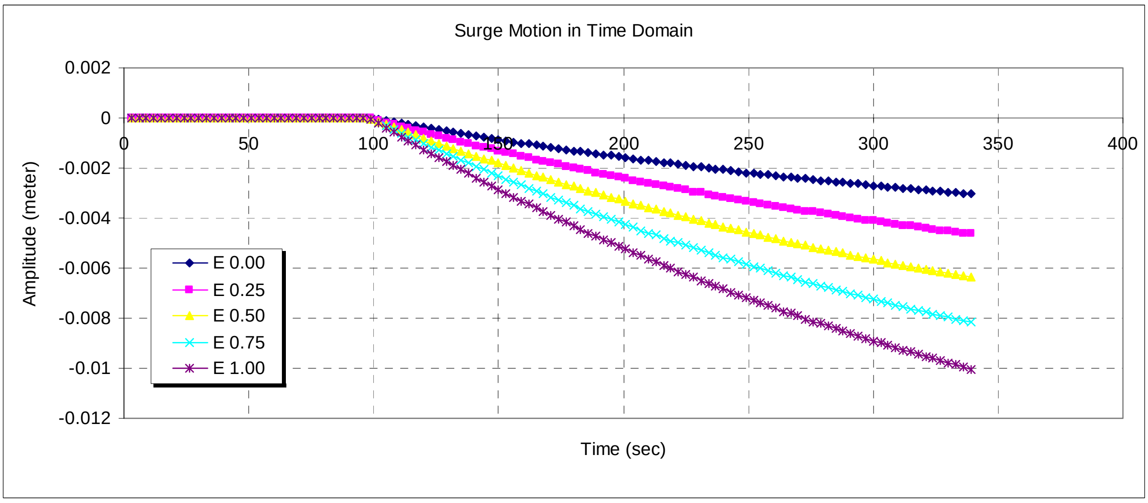


Figure 4.40: Surge motion for various restitutions (struck at 1 knot).

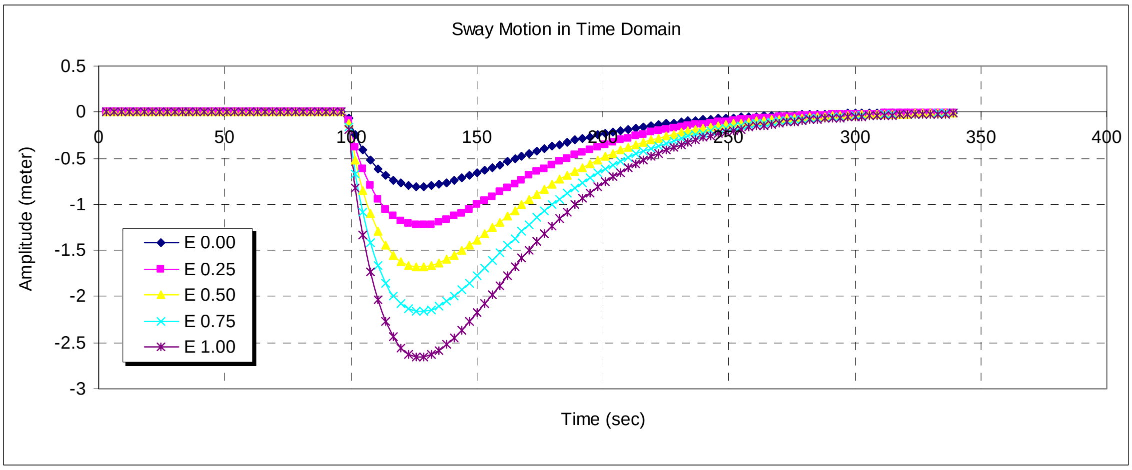


Figure 4.41: Sway motion for various restitutions (struck at 1 knot).

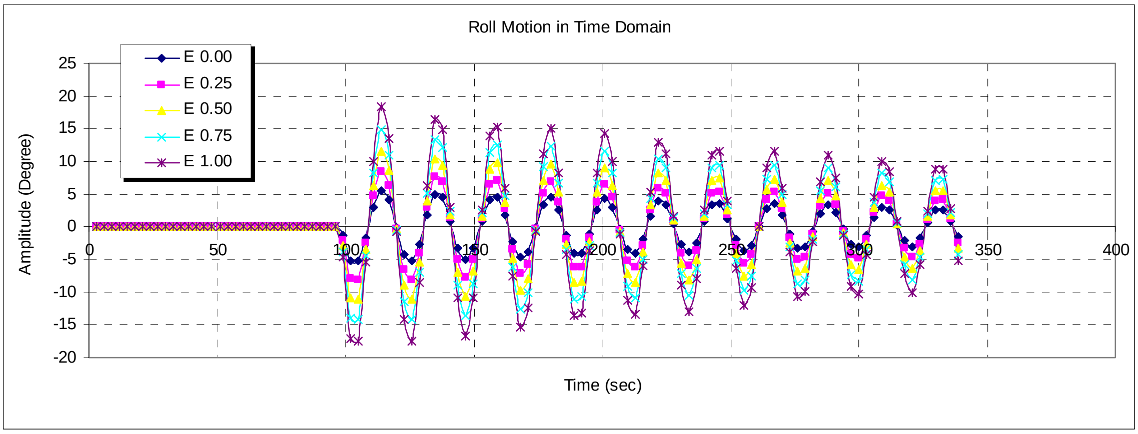


Figure 4.42: Roll motion for various restitutions (struck at 1 knot).

4.3.10 Case 10: 32m vs. 32m hits amidship at 45° at speed 1 knot

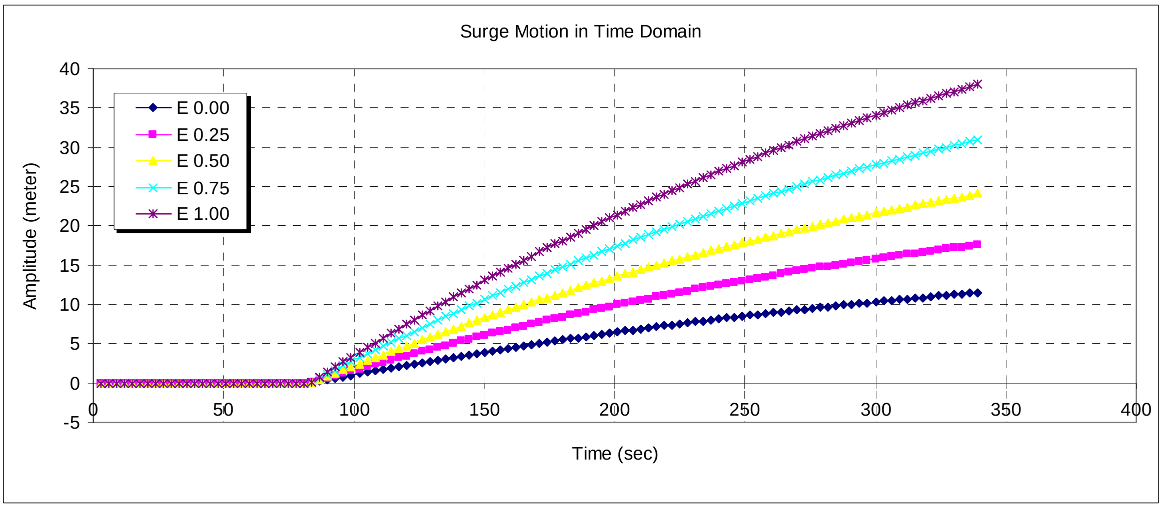


Figure 4.43: Surge motion for various restitutions (struck at 1 knot).

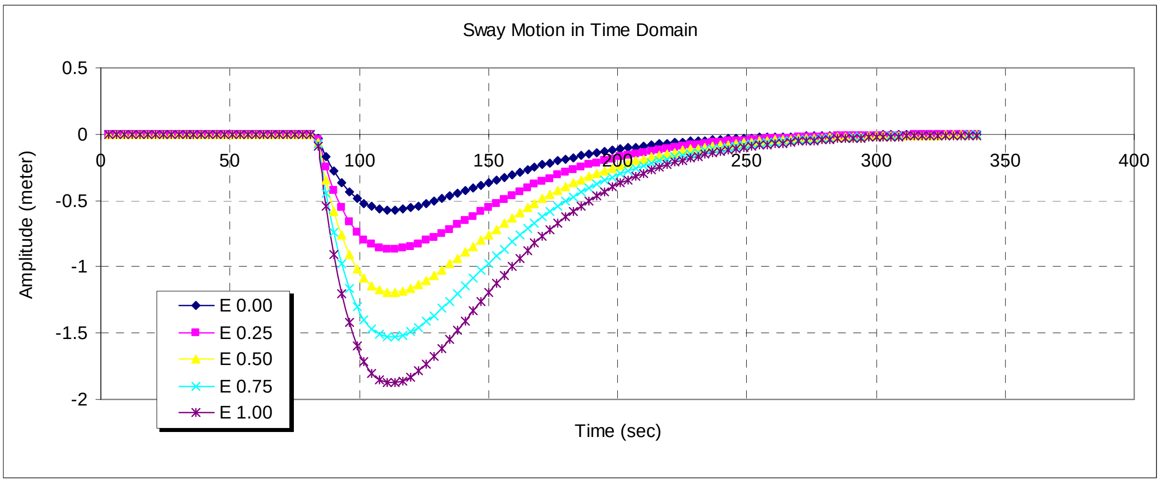


Figure 4.44: Sway motion for various restitutions (struck at 1 knot).

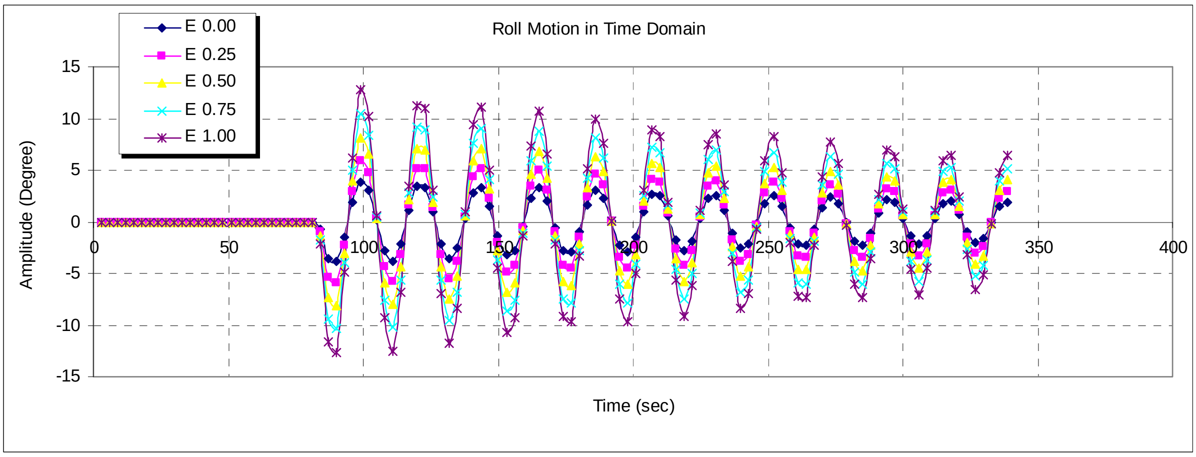


Figure 4.45: Roll motion for various restitutions (struck at 1 knot).

4.3.11 Case 11: 32m vs. 32m at L/4 aft midship at 90º at speed 1 knot

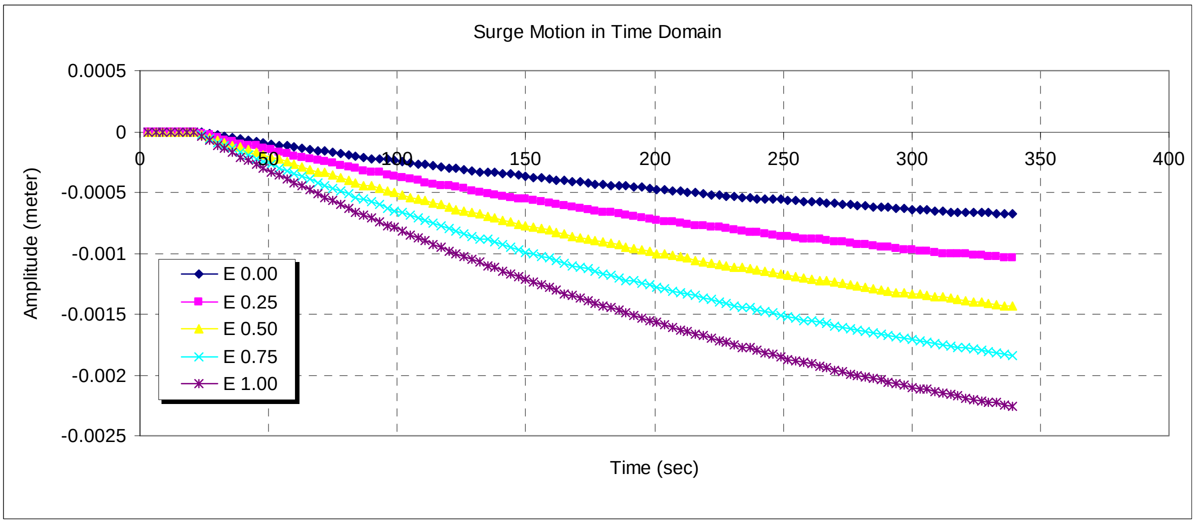


Figure 4.46: Surge motion for various restitutions (struck at 1 knot).

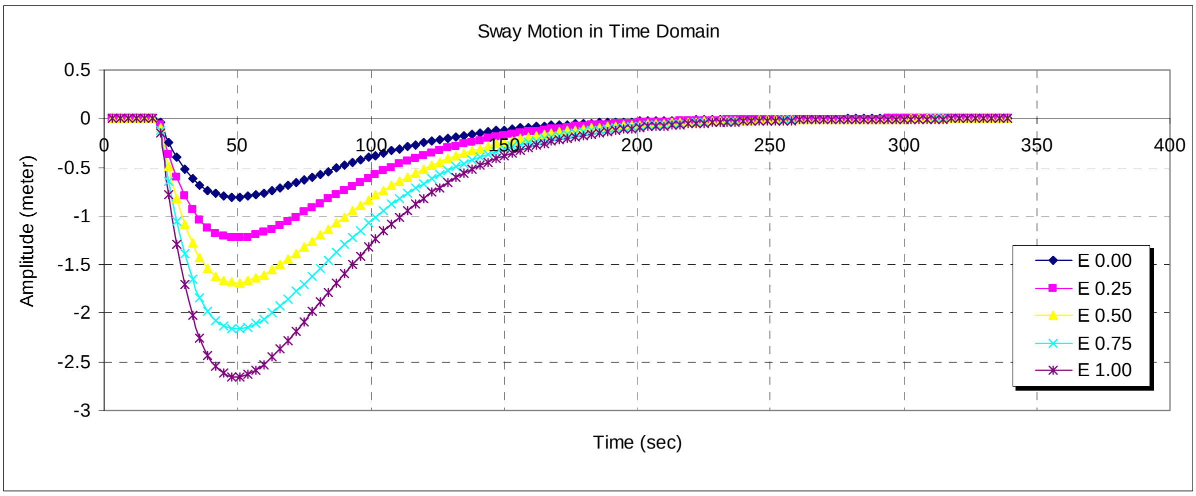


Figure 4.47: Sway motion for various restitutions (struck at 1 knot).

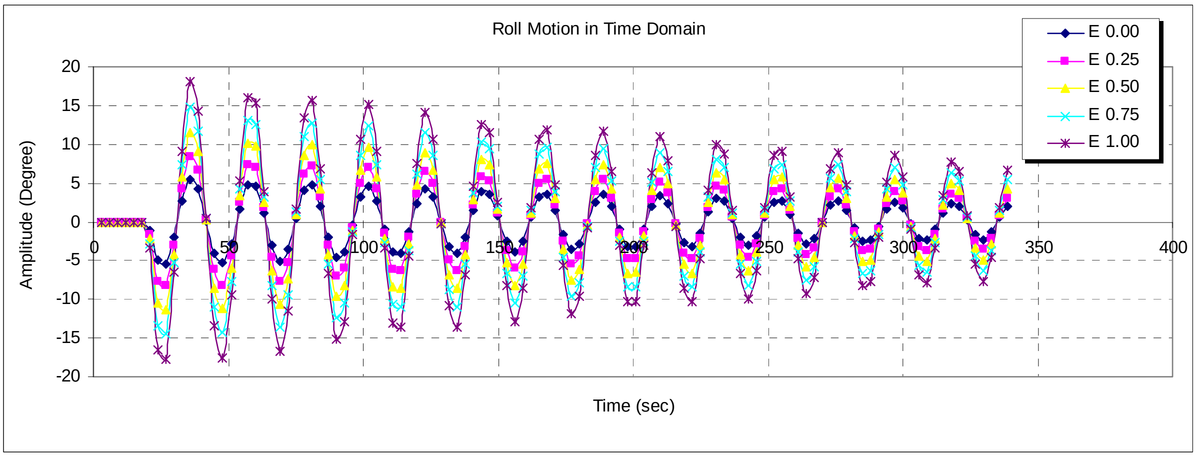


Figure 4.48: Roll motion for various restitutions (struck at 1 knot).

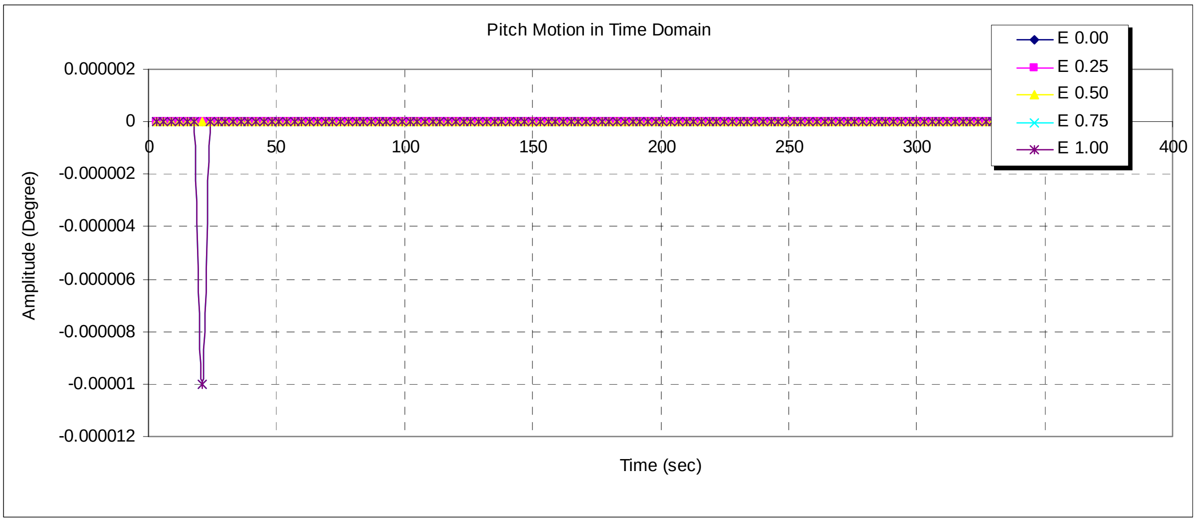


Figure 4.49: Pitch motion for various restitutions (struck at 1 knot).

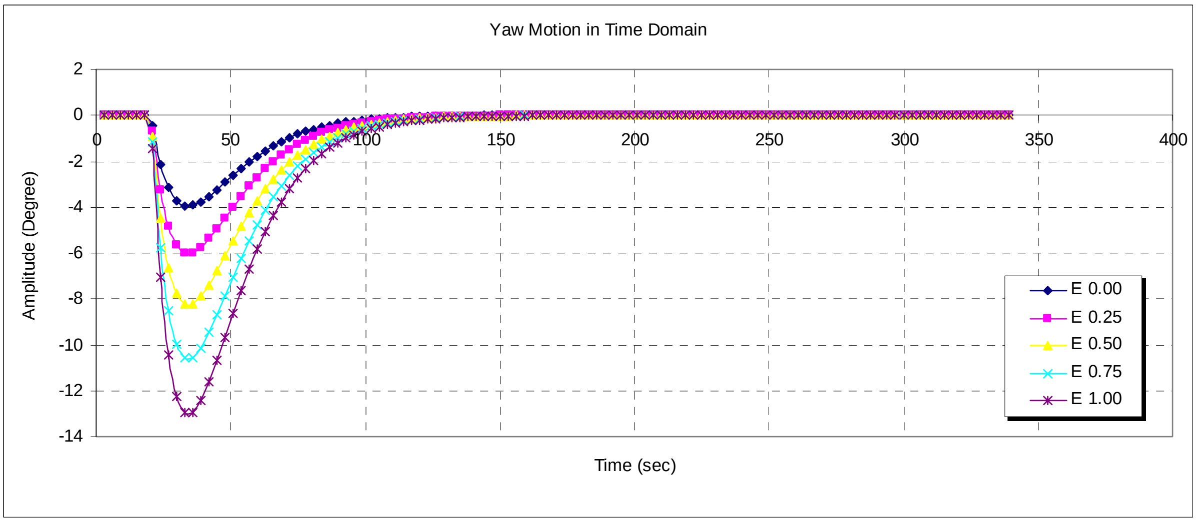


Figure 4.50: Yaw motion for various restitutions (struck at 1 knot).

4.3.12 Case 12: 32m vs. 32m at L/4 aft midship at 45 ° at speed 1 knot

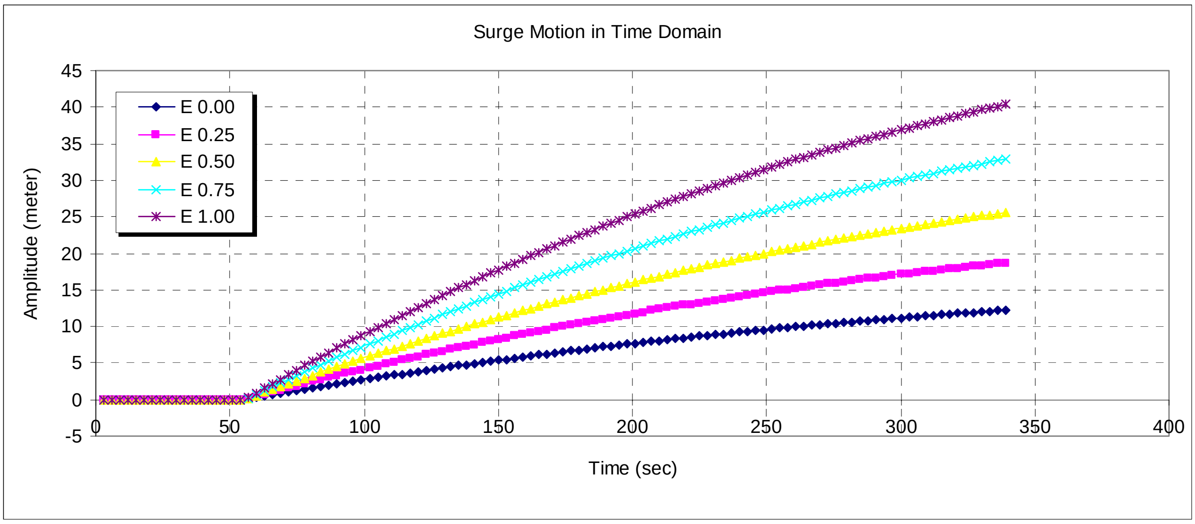


Figure 4.51: Surge motion for various restitutions (struck at 1 knot).

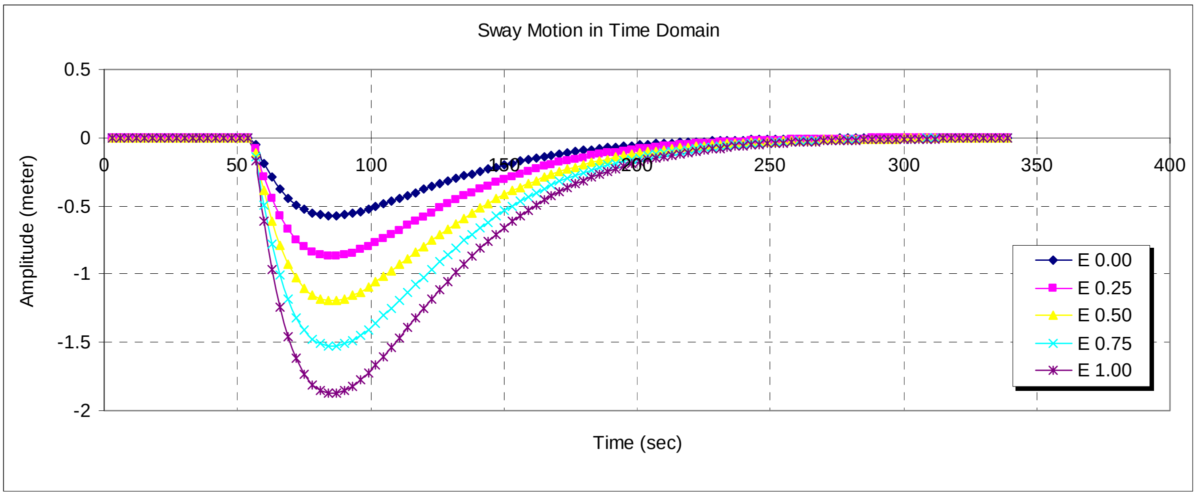


Figure 4.52: Sway motion for various restitutions (struck at 1 knot).

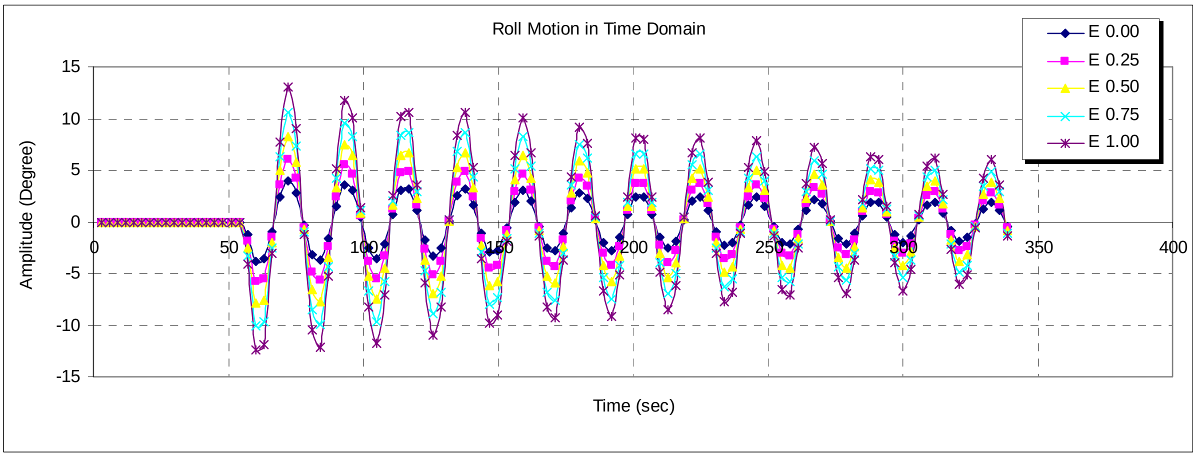


Figure 4.52: Roll motion for various restitutions (struck at 1 knot).

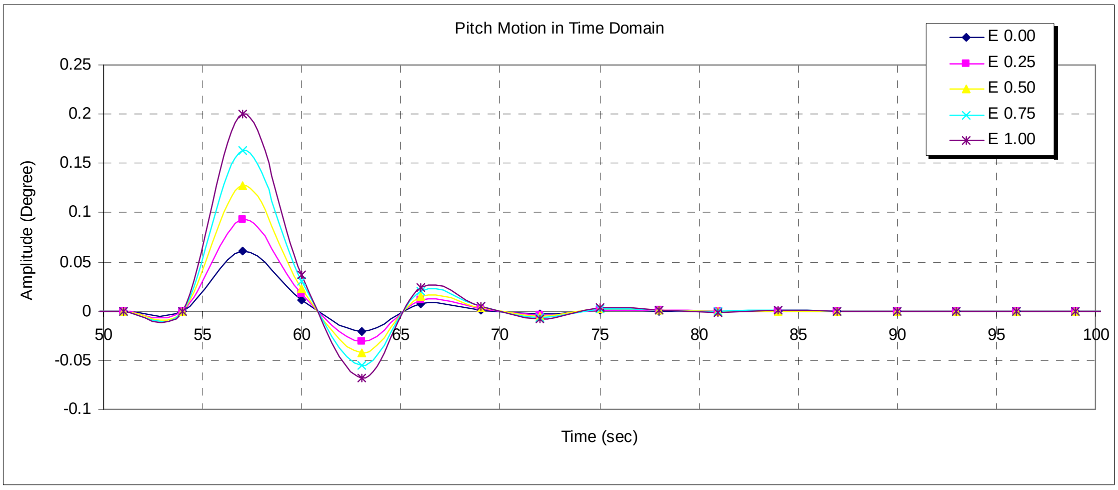


Figure 4.53: Pitch motion for various restitutions (struck at 1 knot).

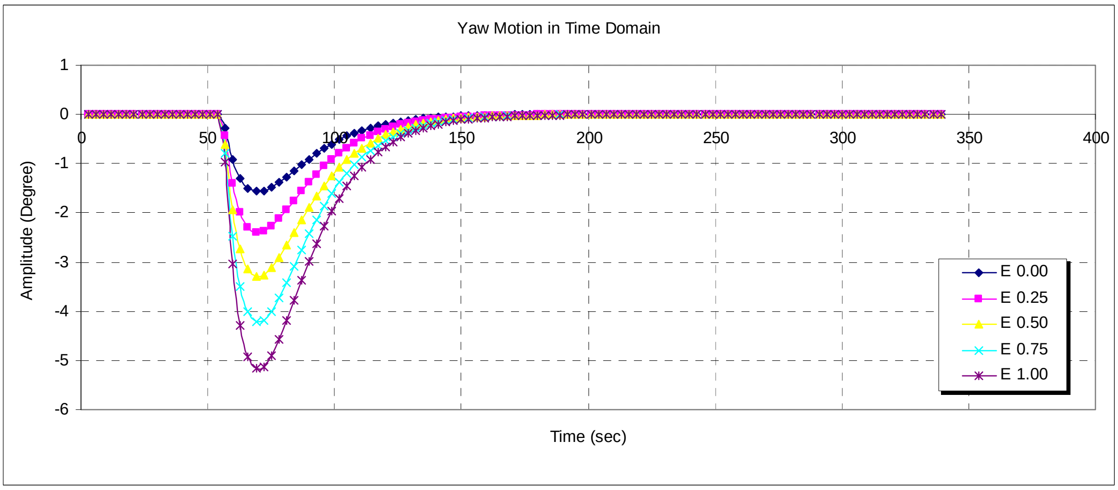


Figure 4.54: Yaw motion for various restitutions (struck at 1 knot).

4.3.13 Case 13: 32m vs. 32m hits amidship at 90º at speed 6 knot

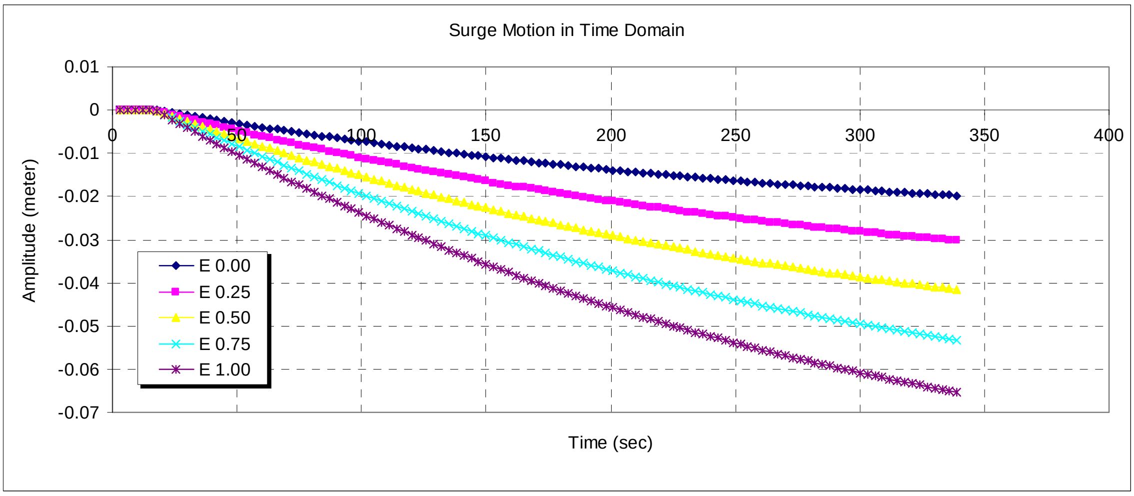


Figure 4.55: Surge motion for various restitutions (struck at 6 knot).

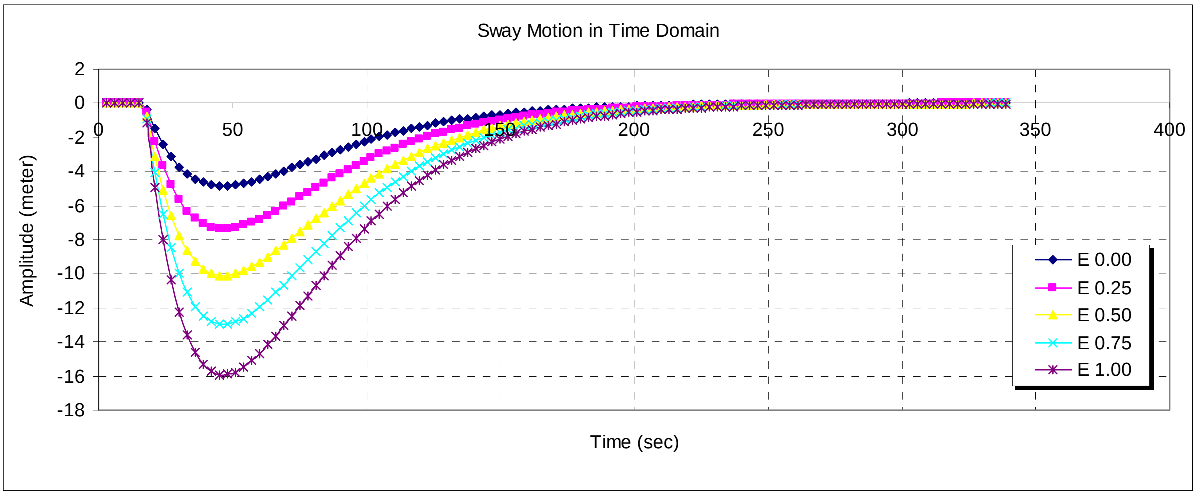


Figure 4.56: Sway motion for various restitutions (struck at 6 knot).

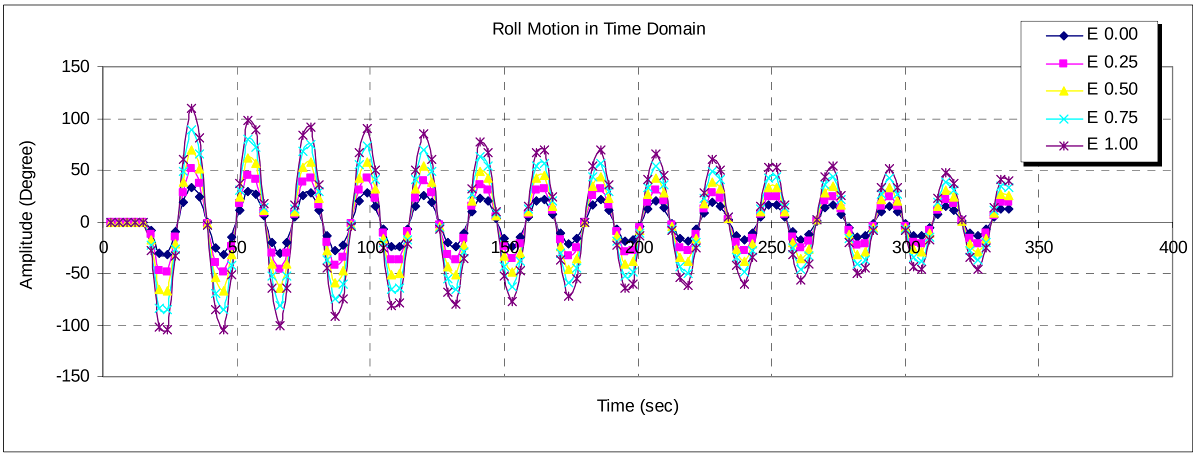


Figure 4.57: Roll motion for various restitutions (struck at 6 knot).

4.3.14 Case 14: 32m vs. 32m hits amidship at 45° at speed 6 knot

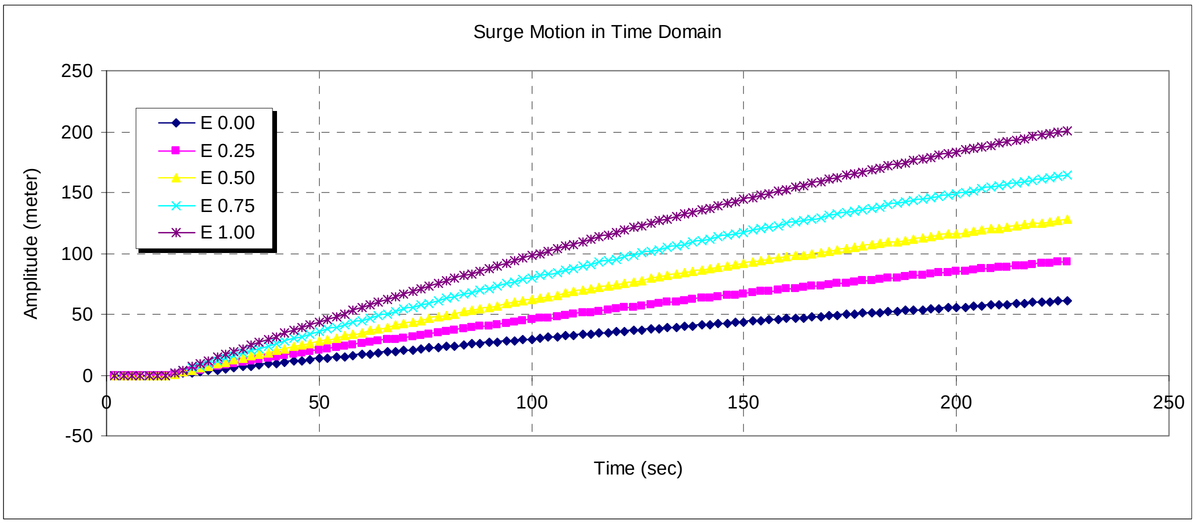


Figure 4.58: Surge motion for various restitutions (struck at 6 knot).

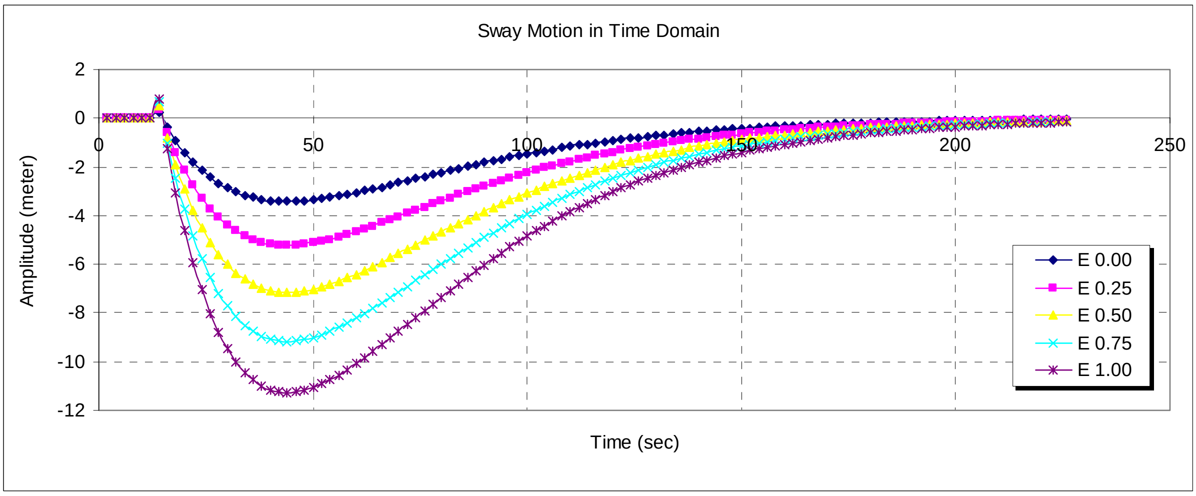


Figure 4.59: Sway motion for various restitutions (struck at 6 knot).

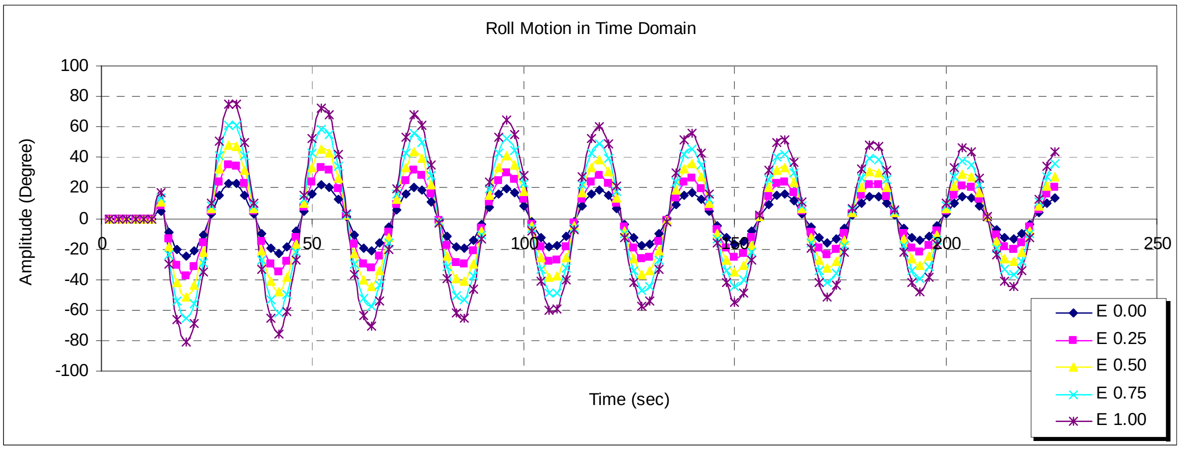


Figure 4.60: Roll motion for various restitutions (struck at 6 knot).

4.3.15 Case 15: 32m vs. 32m at L/4 aft midship at 90° at speed 6 knot

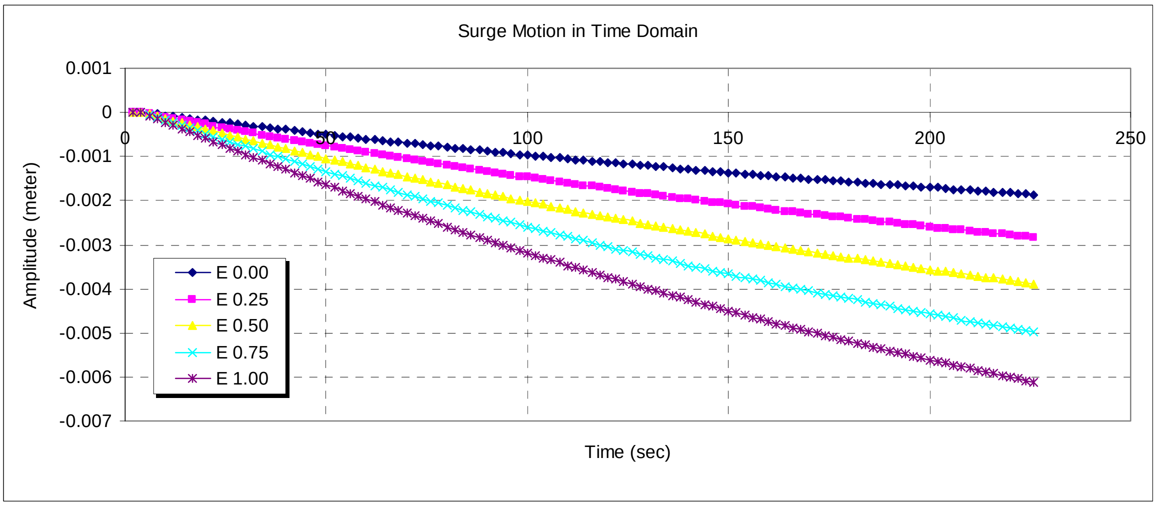


Figure 4.61: Surge motion for various restitutions (struck at 6 knot).

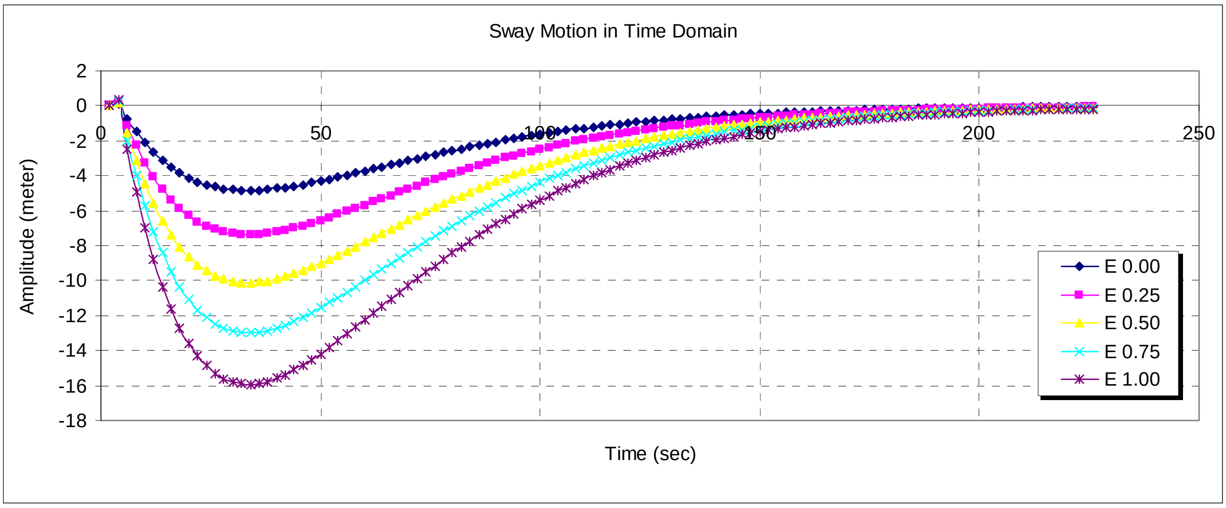


Figure 4.62: Sway motion for various restitutions (struck at 6 knot).

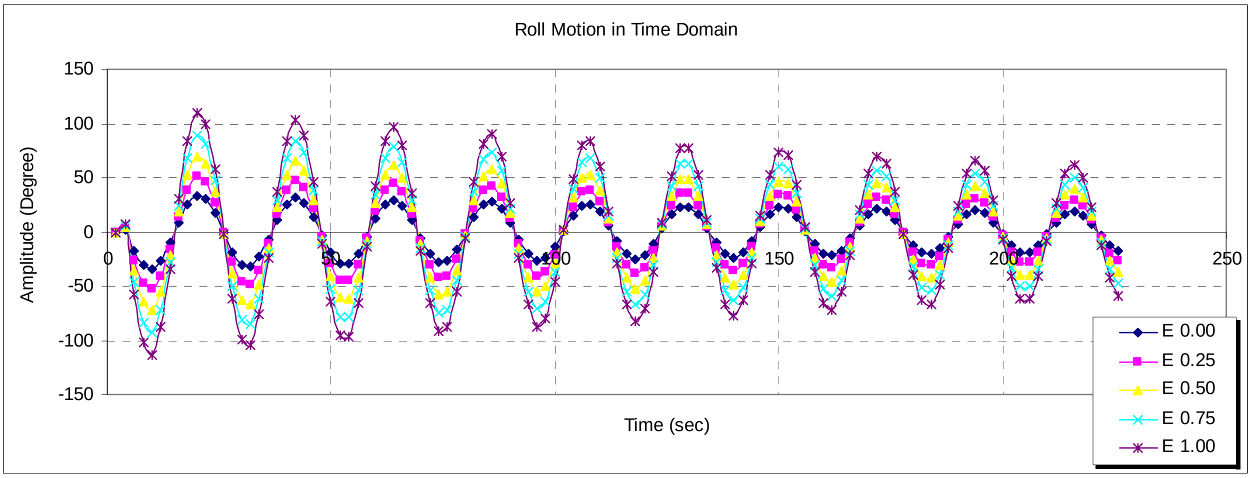


Figure 4.63: Roll motion for various restitutions (struck at 6 knot).

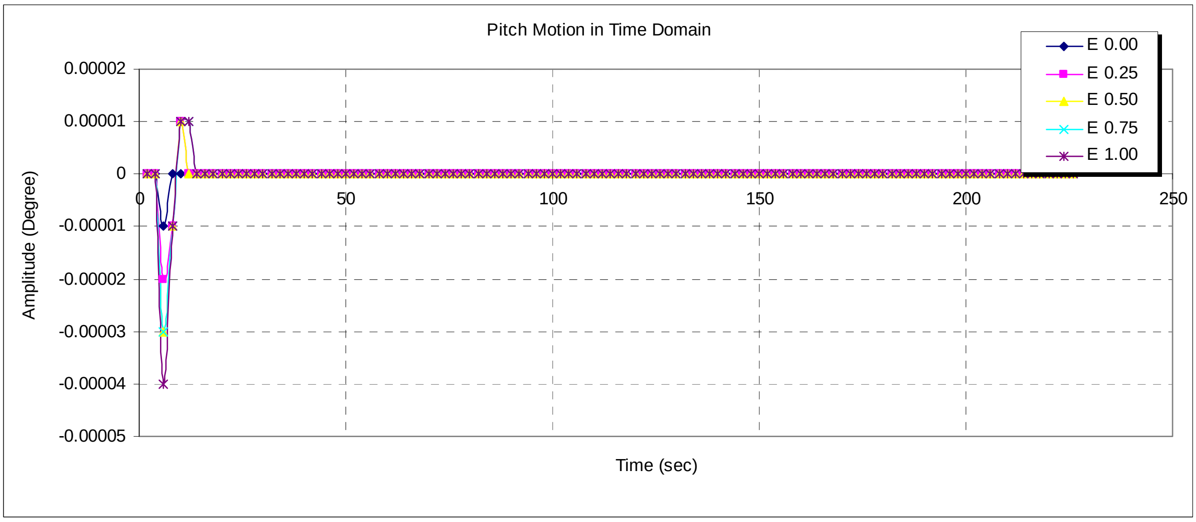


Figure 4.64: Roll motion for various restitutions (struck at 6 knot).

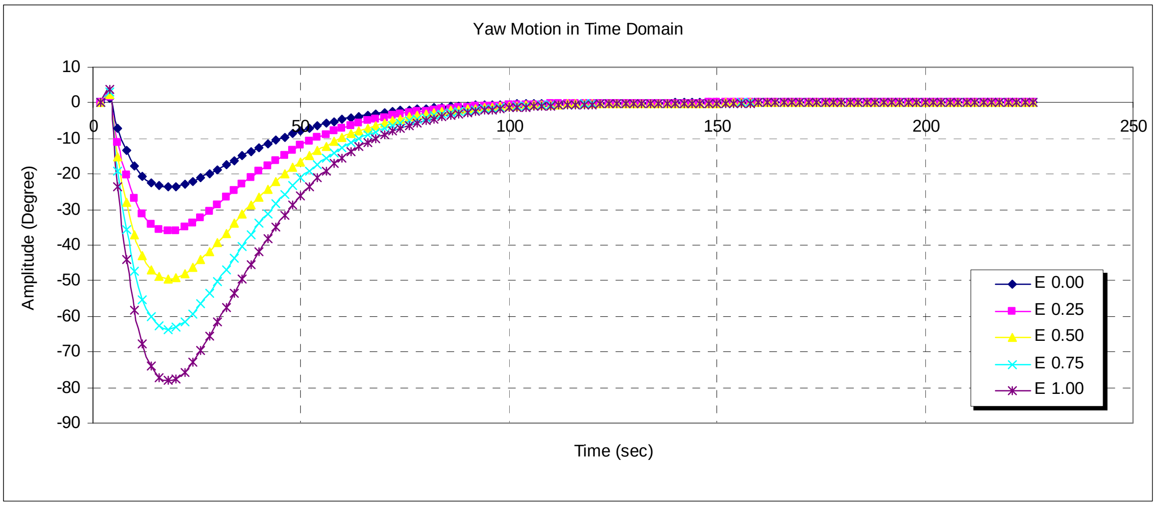


Figure 4.65: Yaw motion for various restitutions (struck at 6 knot).

4.3.16 Case 16: 32m vs. 32m at L/4 aft midship at 45 ° at speed 6 knot

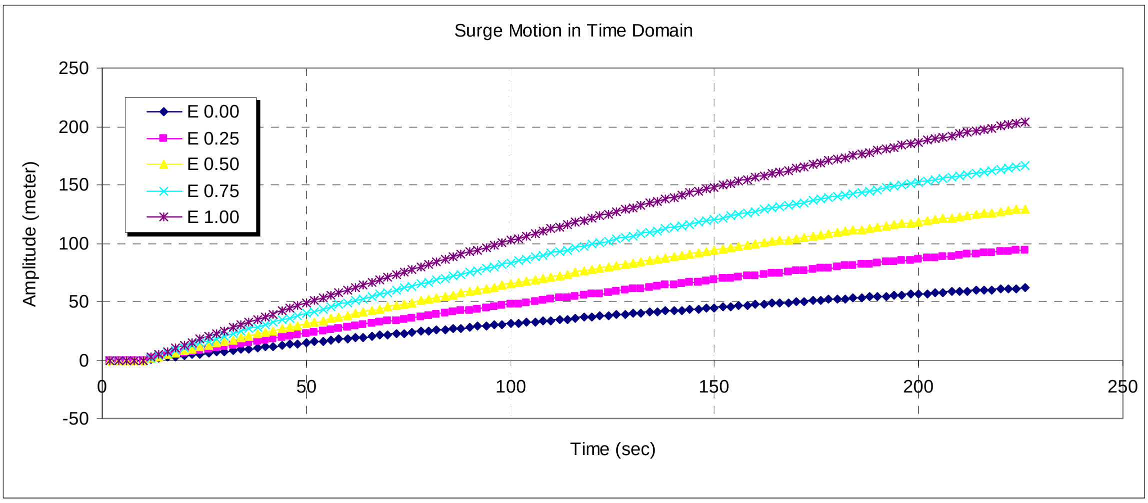


Figure 4.66: Surge motion for various restitutions (struck at 6 knot).

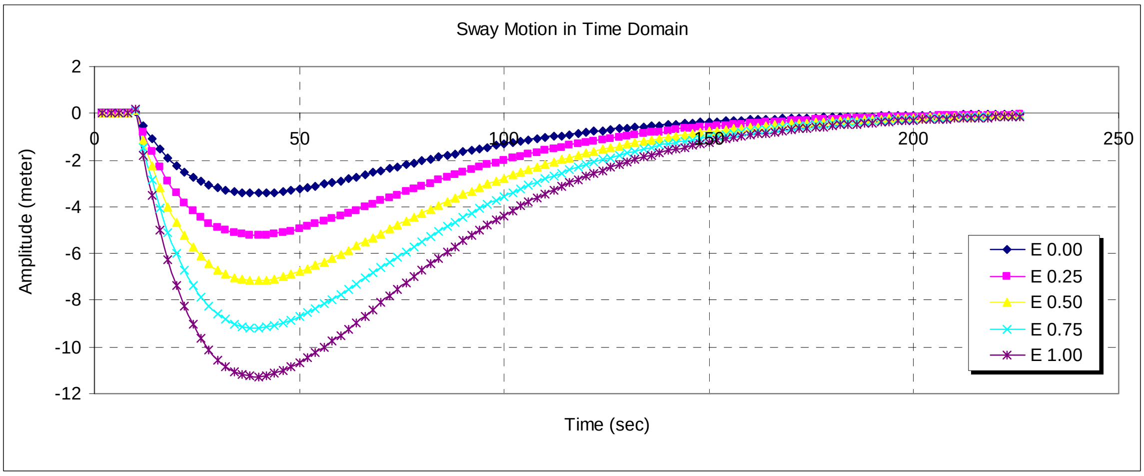


Figure 4.67: Sway motion for various restitutions (struck at 6 knot).

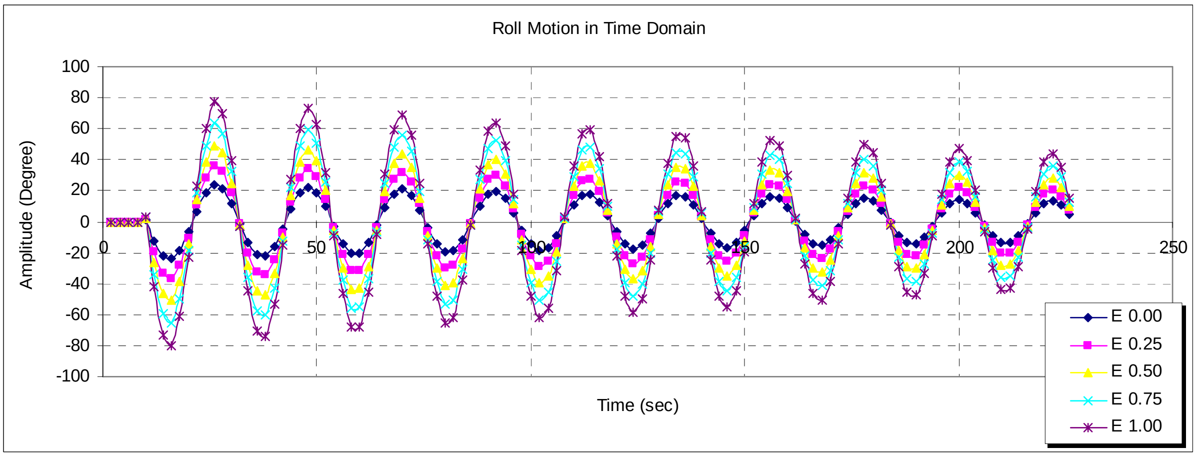


Figure 4.68: Roll motion for various restitutions (struck at 6 knot).

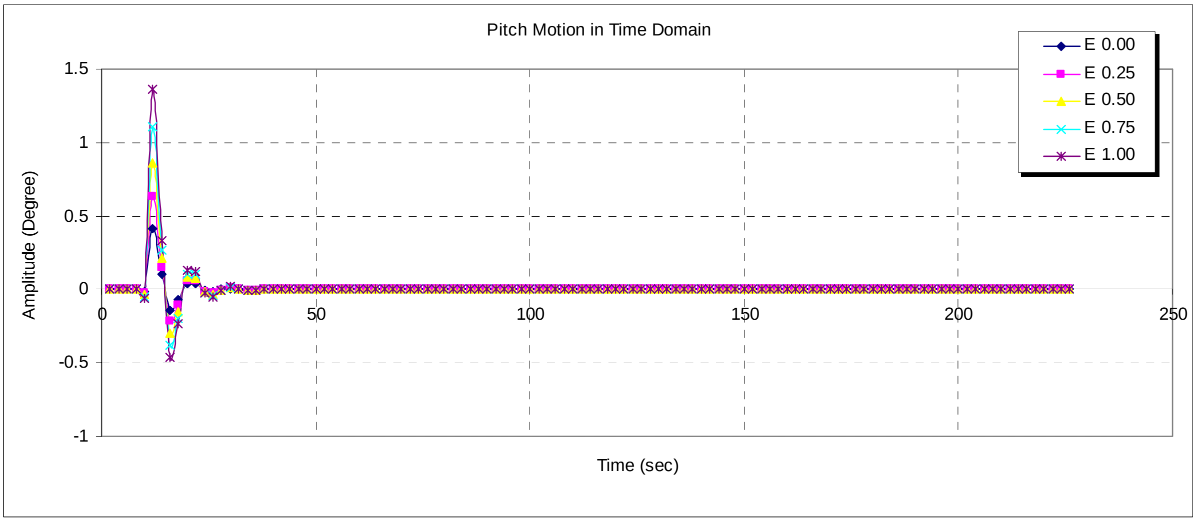


Figure 4.69: Pitch motion for various restitutions (struck at 6 knot).

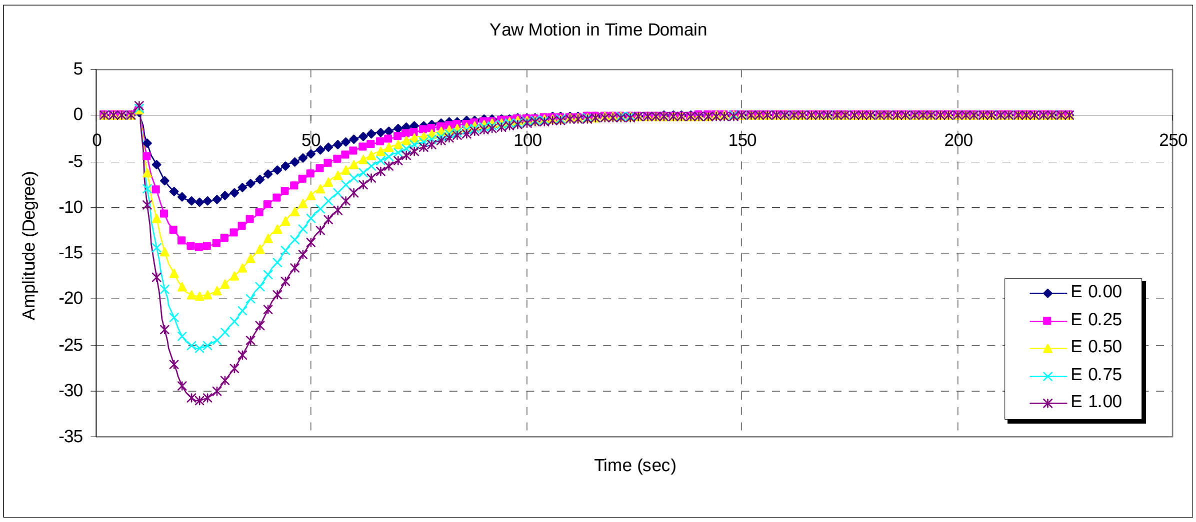


Figure 4.70: Yaw motion for various restitutions (struck at 6 knot).

4.3.17 Case 17: 32m vs. 46m at midship at 90 ° at speed 1 knot

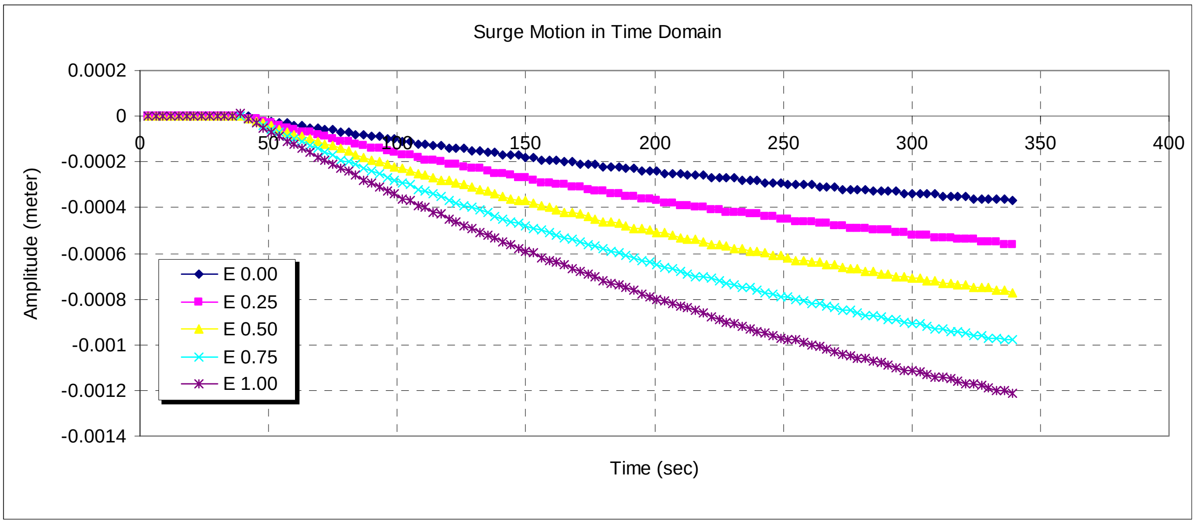


Figure 4.71: Surge motion for various restitutions (struck at 1 knot).

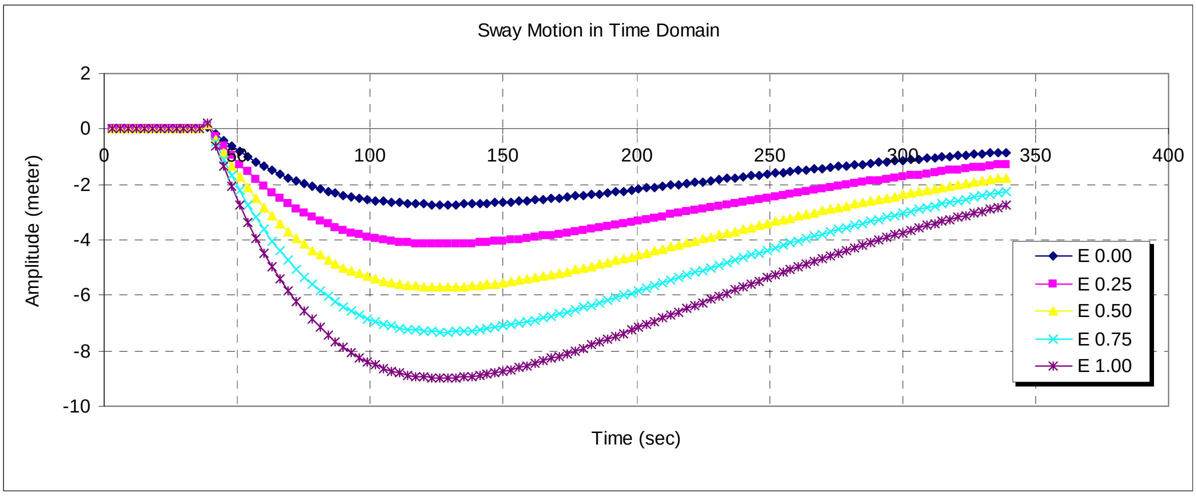


Figure 4.72: Sway motion for various restitutions (struck at 1 knot).

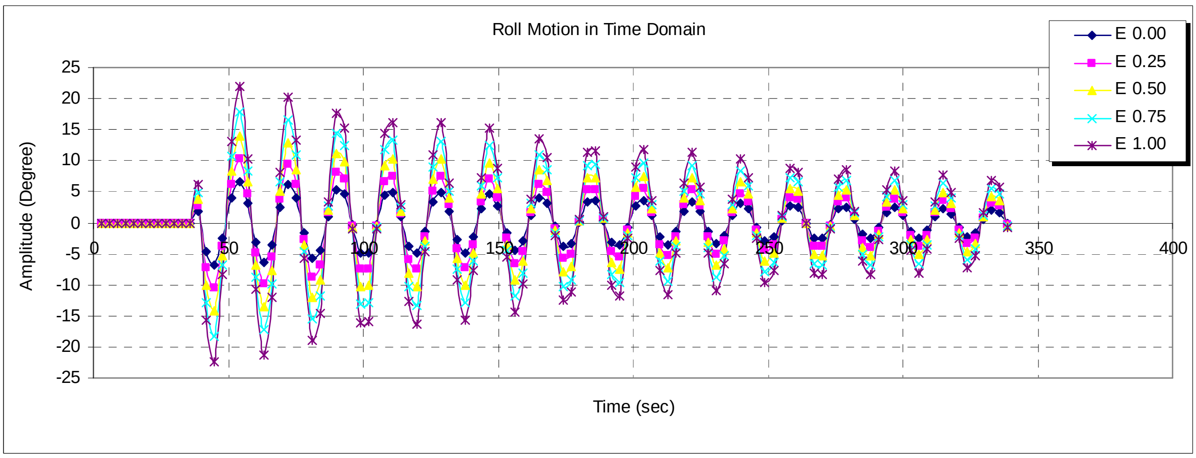


Figure 4.73: Roll motion for various restitutions (struck at 1 knot).

4.3.18 Case 18: 32m vs. 46m at midship at 90 ° at speed 3 knot

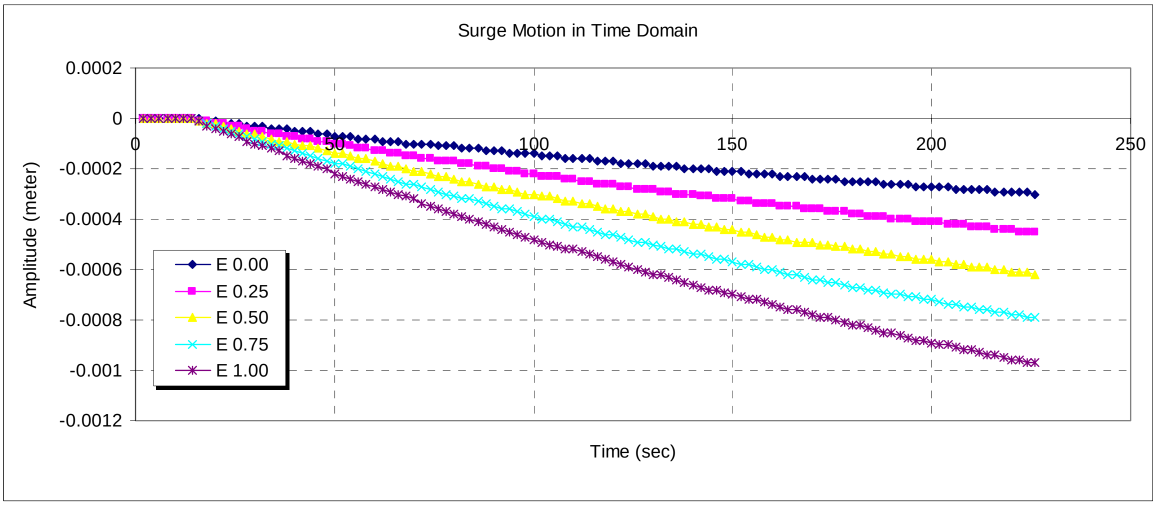


Figure 4.74: Surge motion for various restitutions (struck at 3 knot).

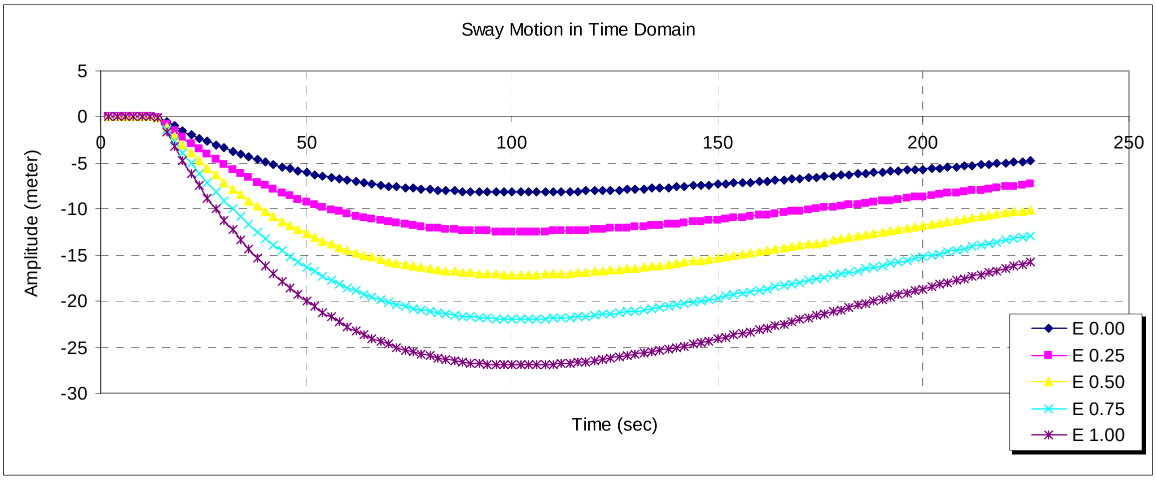


Figure 4.75: Sway motion for various restitutions (struck at 3 knot).

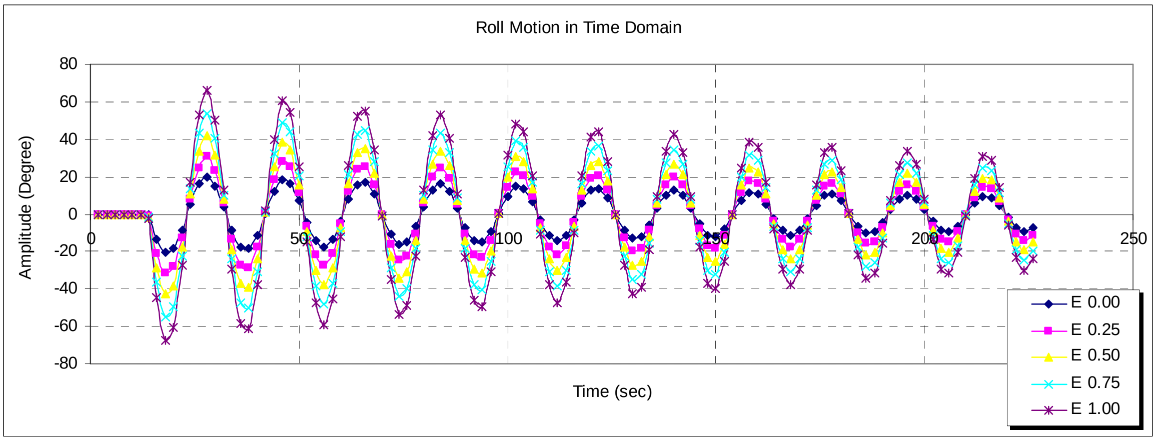


Figure 4.76: Roll motion for various restitutions (struck at 3 knot).

4.3.19 Case 19: 32m vs. 46m at midship at 90 ° at speed 6 knot

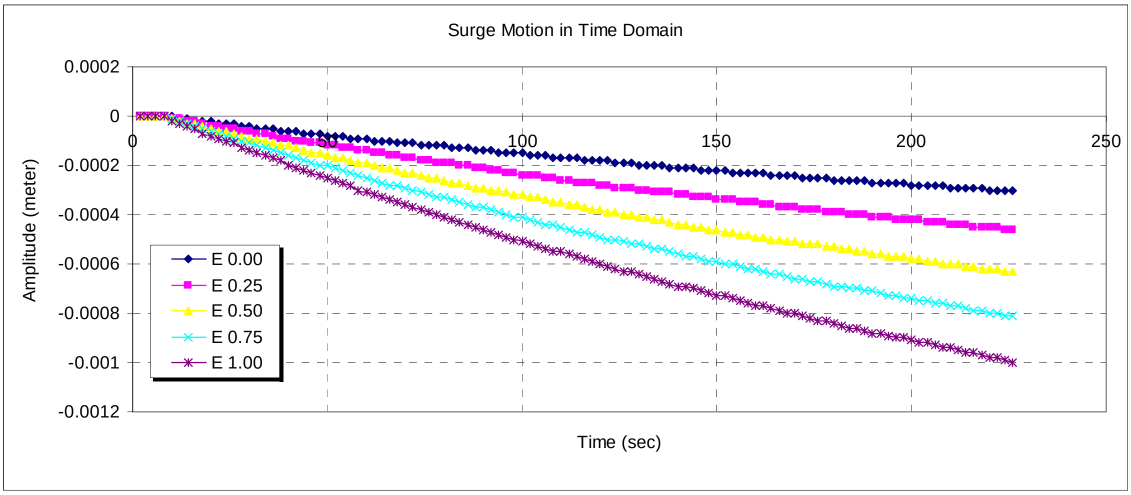


Figure 4.77: Surge motion for various restitutions (struck at 6 knot).

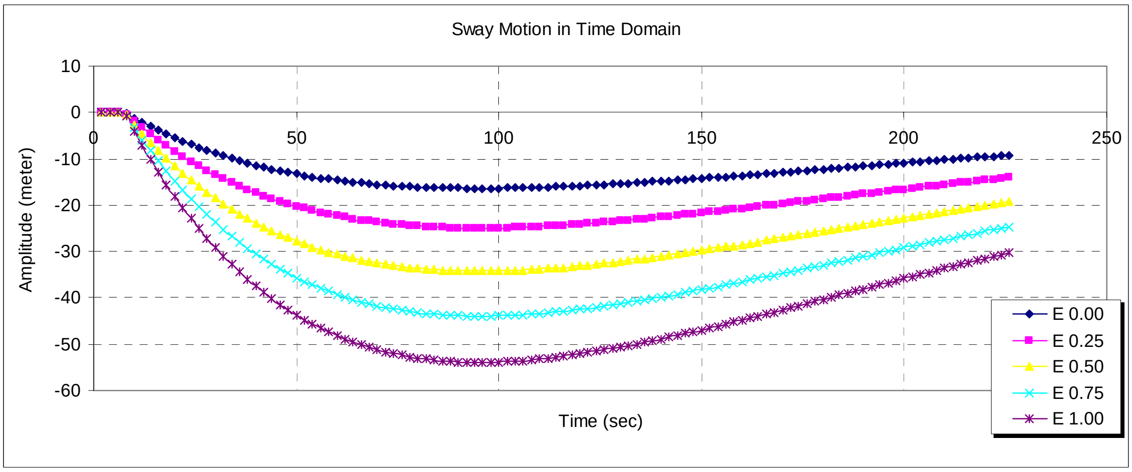


Figure 4.78: Sway motion for various restitutions (struck at 6 knot).

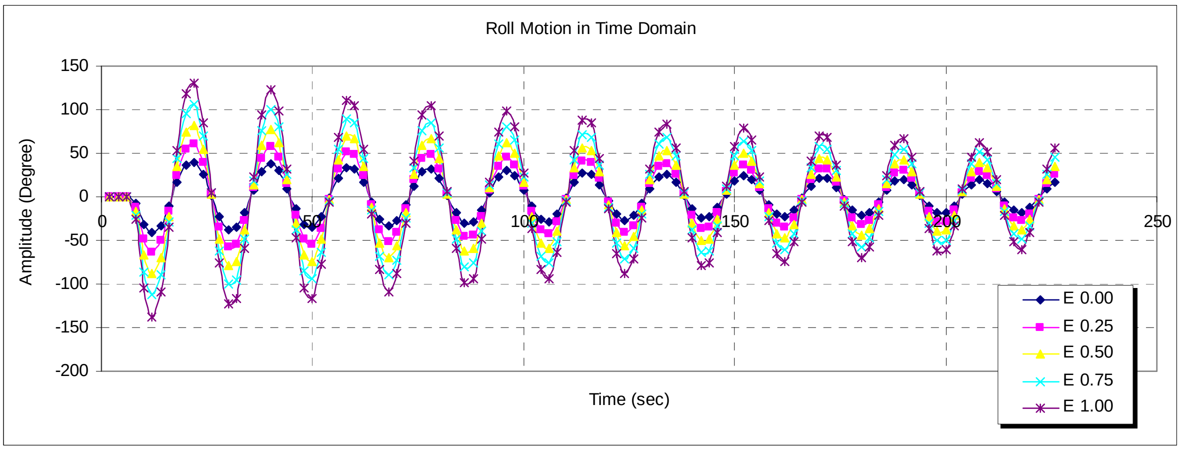


Figure 4.79: Roll motion for various restitutions (struck at 6 knot).

4.3.20 Case 20: 32m vs. 46m at L/4 aft of midship at 90 ° at speed 1 knot

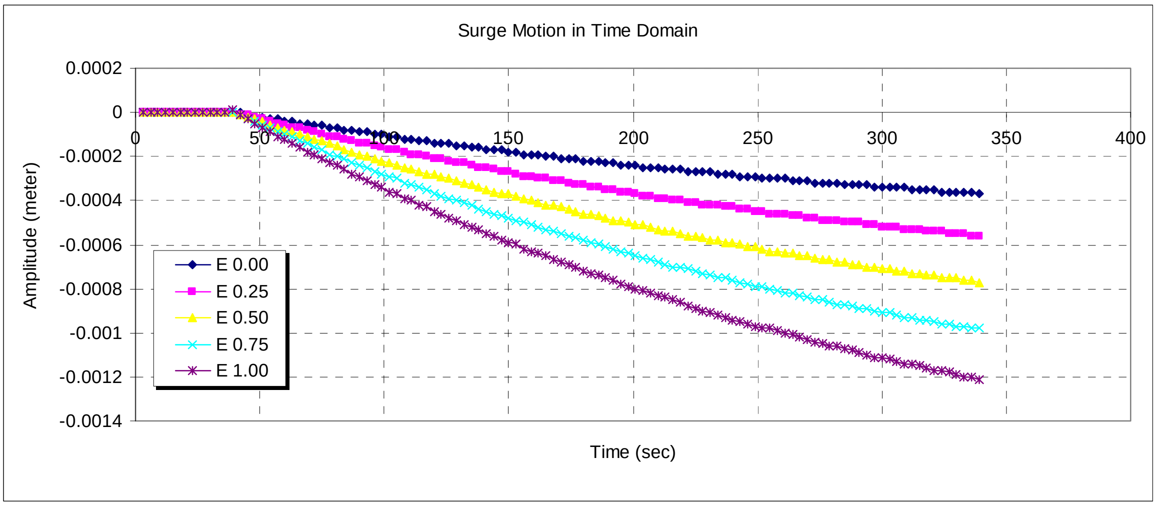


Figure 4.80: Surge motion for various restitutions (struck at 1 knot).

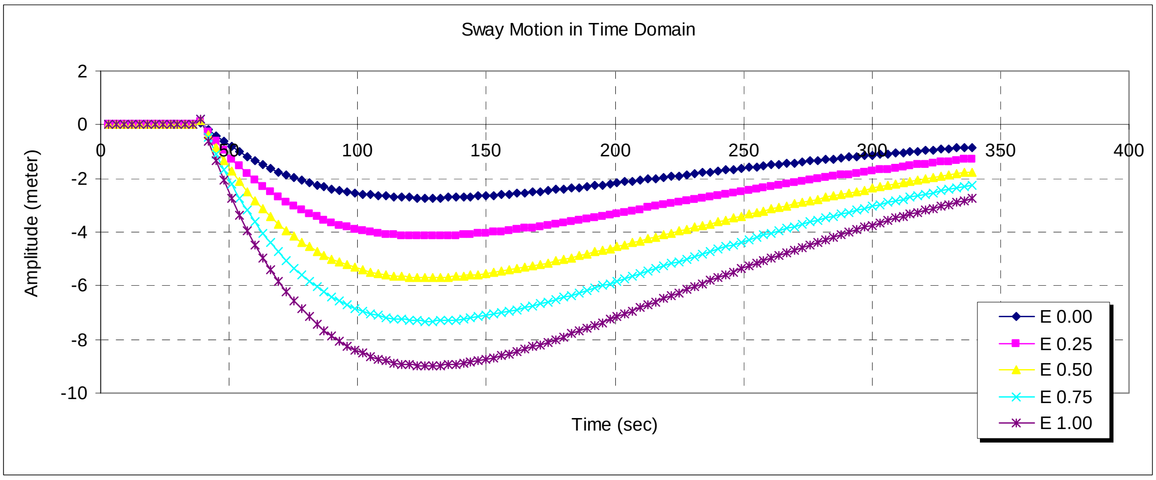


Figure 4.81: Sway motion for various restitutions (struck at 1 knot).

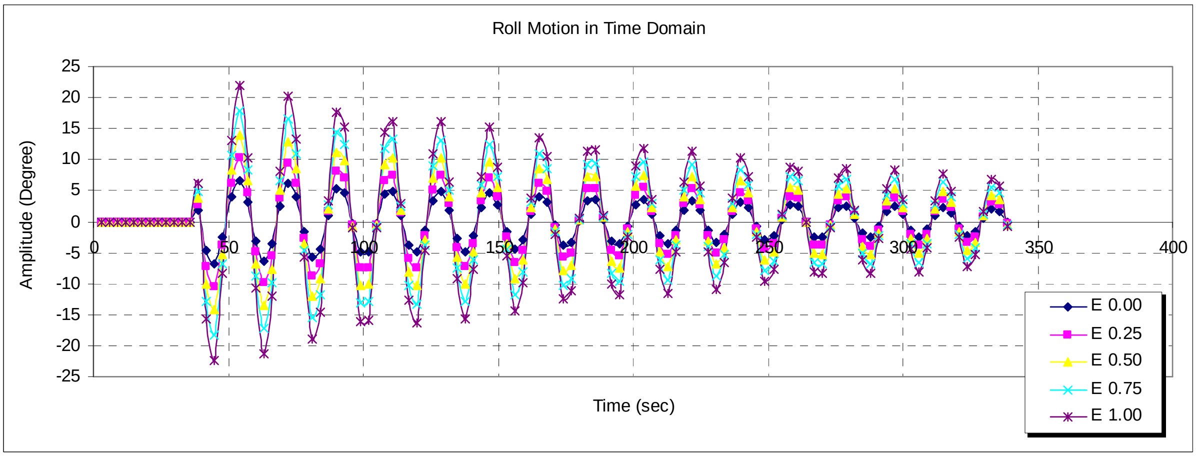


Figure 4.82: Roll motion for various restitutions (struck at 1 knot).

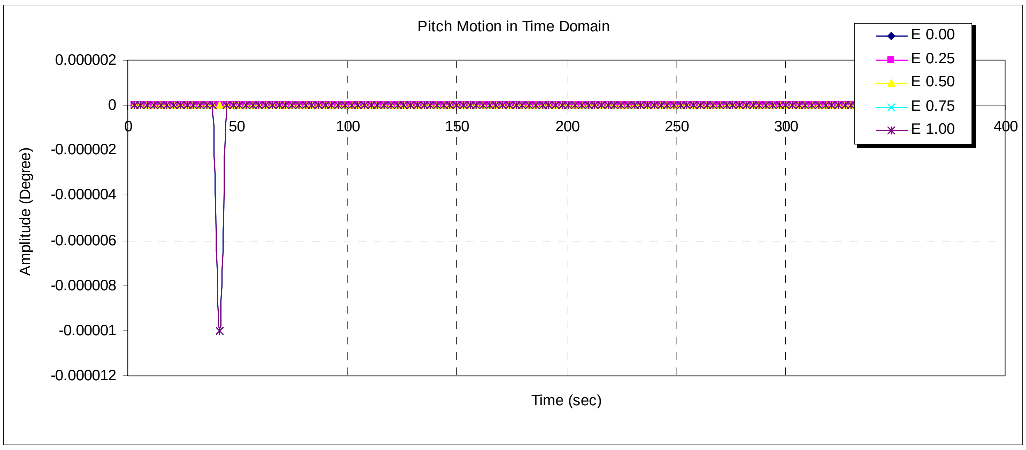


Figure 4.83: Pitch motion for various restitutions (struck at 1 knot).

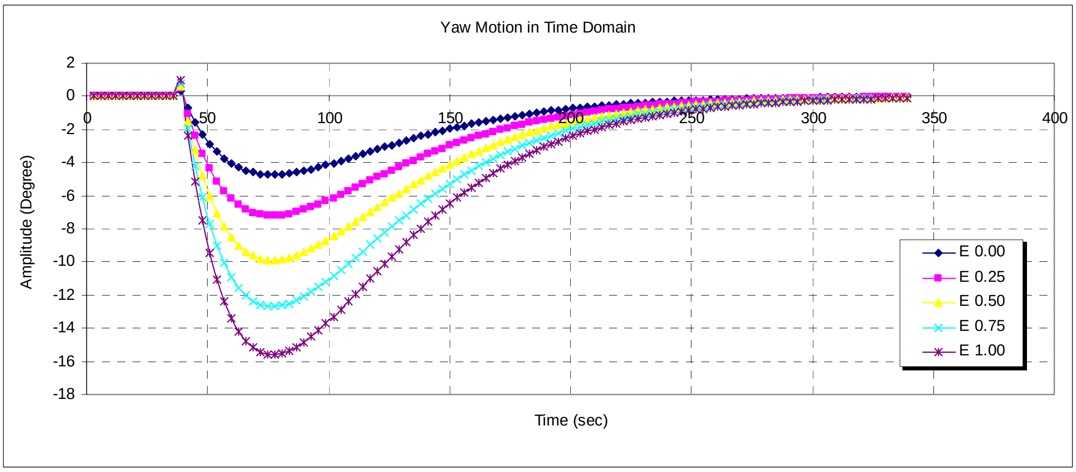


Figure 4.84: Yaw motion for various restitutions (struck at 1 knot).

4.3.21 Case 21: 32m vs. 46m at L/4 aft of midship at 90 ° at speed 3 knot

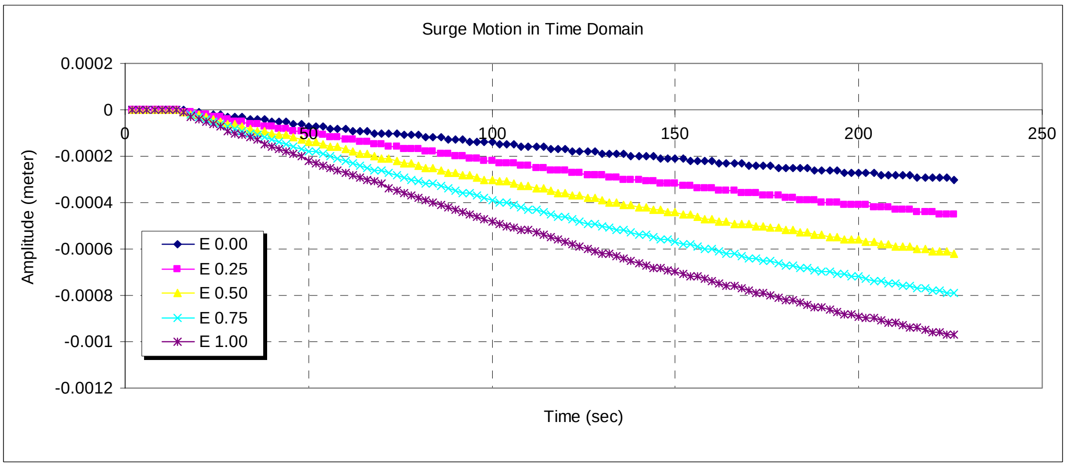


Figure 4.85: Surge motion for various restitutions (struck at 3 knot).

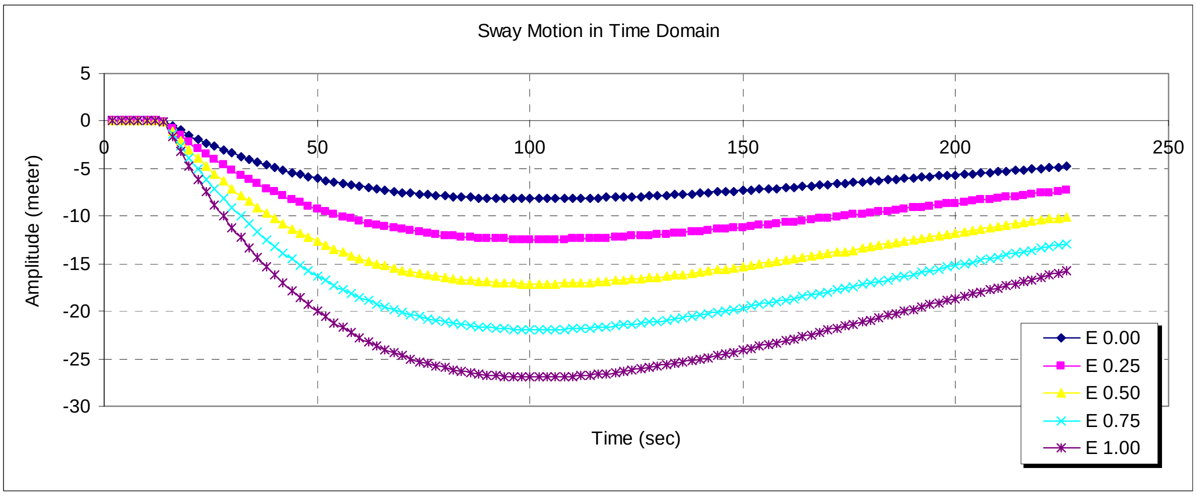


Figure 4.86: Sway motion for various restitutions (struck at 3 knot).

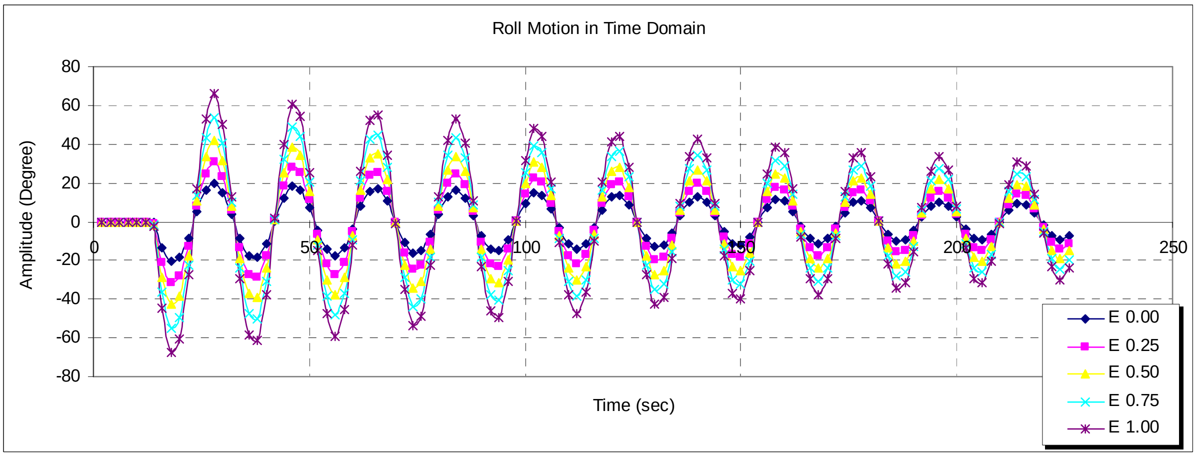


Figure 4.87: Roll motion for various restitutions (struck at 3 knot).

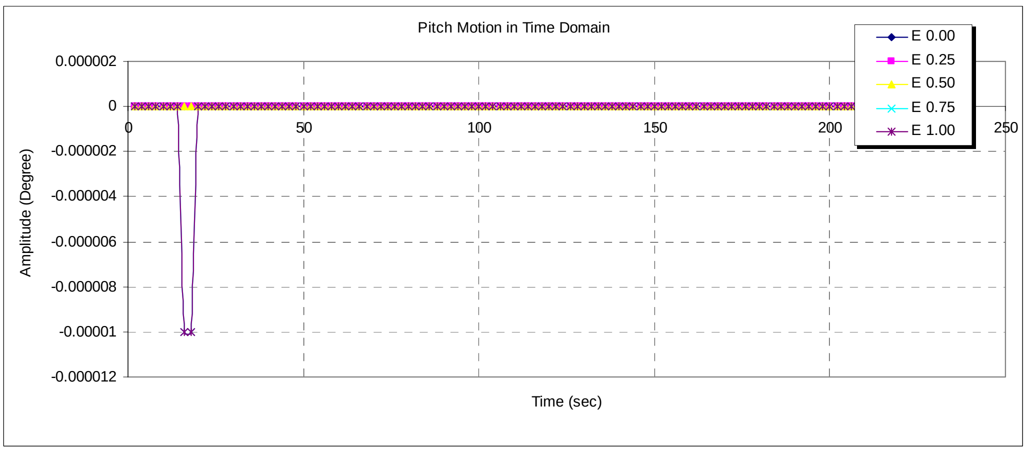


Figure 4.88: Pitch motion for various restitutions (struck at 3 knot).

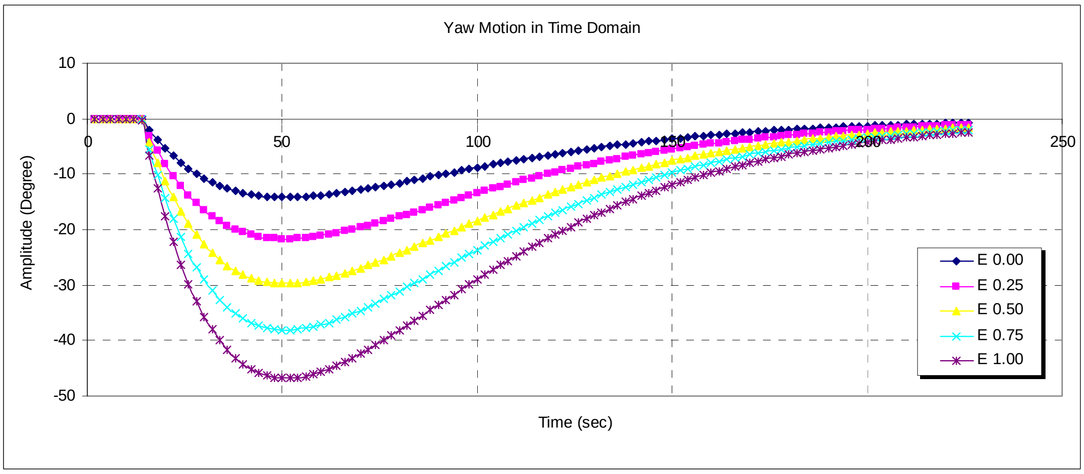


Figure 4.89: Yaw motion for various restitutions (struck at 3 knot).

4.3.22 Case 22: 32m vs. 46m at L/4 aft of midship at 90 º at speed 6 knot

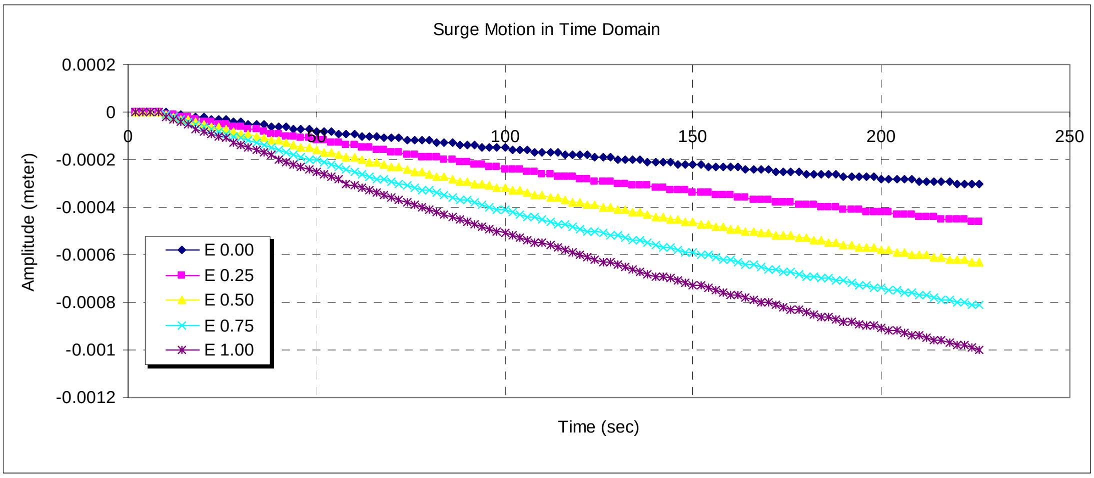


Figure 4.90: Surge motion for various restitutions (struck at 6 knot).

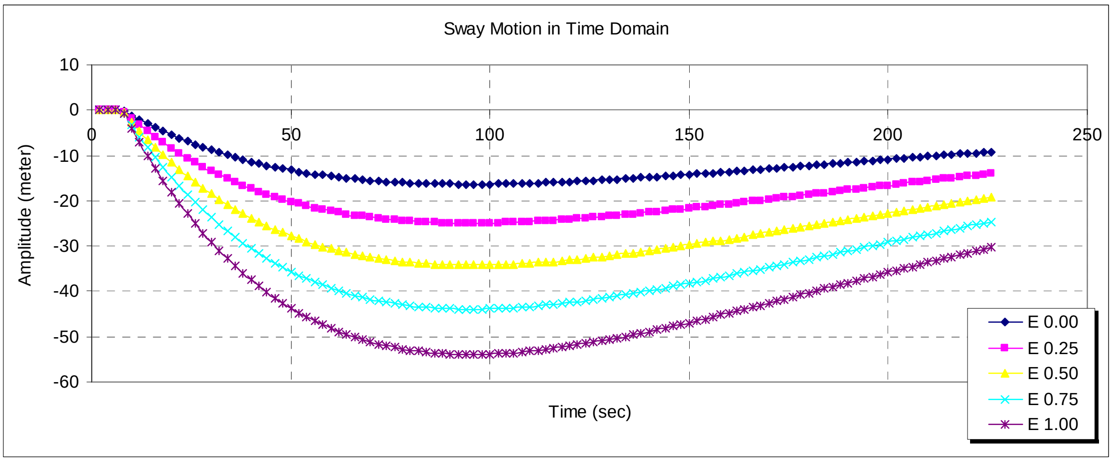


Figure 4.91: Sway motion for various restitutions (struck at 6 knot).

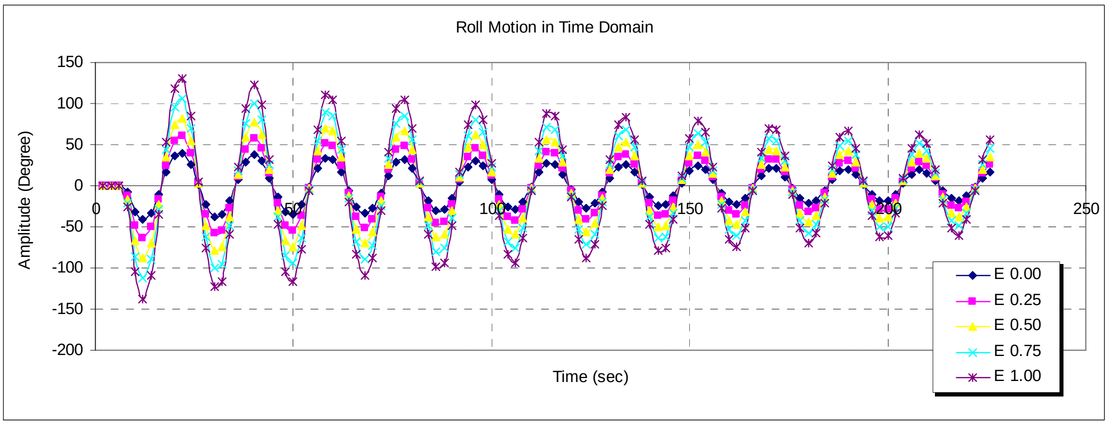


Figure 4.92: Roll motion for various restitutions (struck at 6 knot).

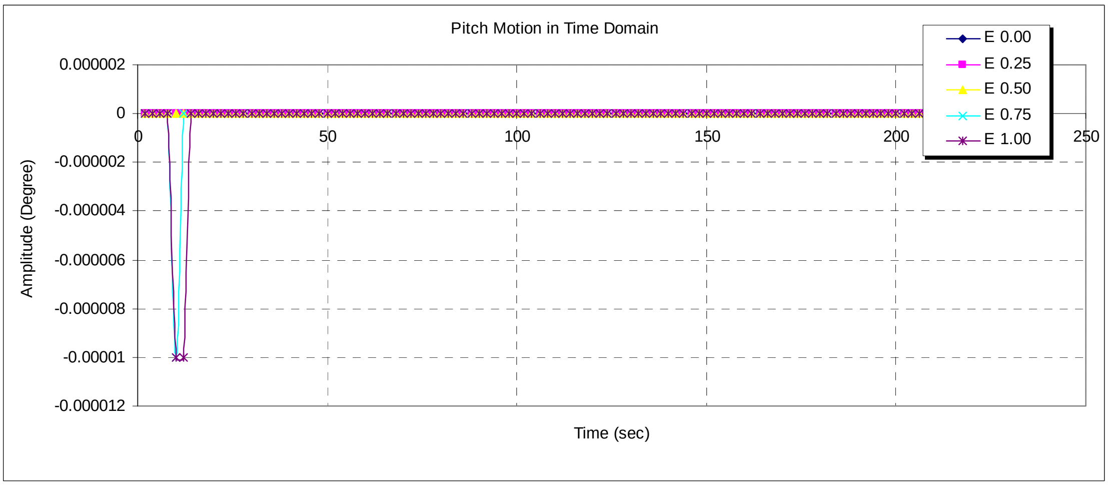


Figure 4.93: Pitch motion for various restitutions (struck at 6 knot).

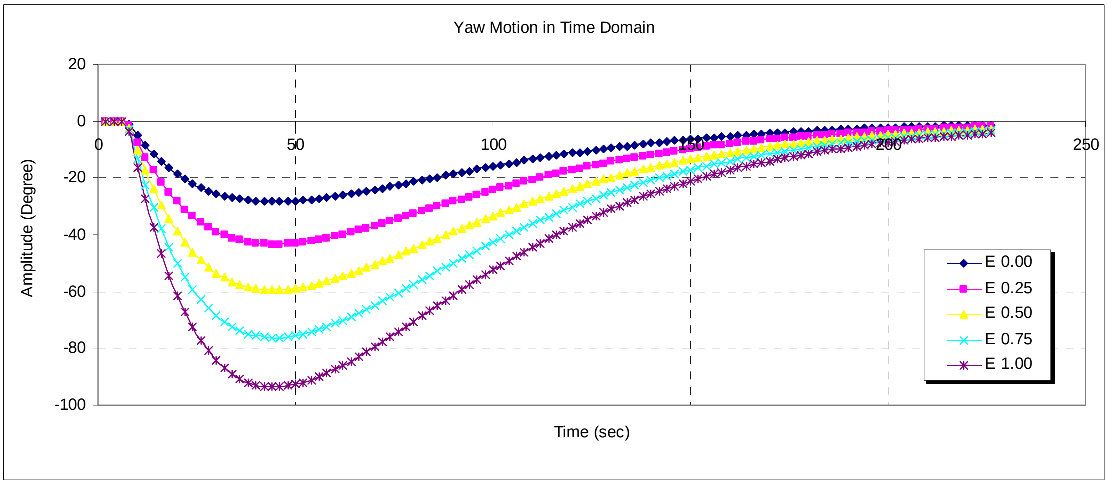


Figure 4.94: Yaw motion for various restitutions (struck at 6 knot).

4.4 Analysis by Altering the Collision Time

It has been observed through studying the results of the Cases 3 to 8 that the amplitudes of motions, particularly for sway and yaw, are very large in comparison to the vessel's length. Although the amplitudes may be observed to be very large for coefficient of restitution one, the reason may be due to the very small collision time (1 second in these cases). Therefore, in this section further analysis is being conducted considering various collision times. Figure 4.95, 4.96, 4.97 and 4.98 represents the surge, sway, roll and yaw motions respectively for Case 3 at different collision times (ranging 1 second to 5 seconds). The coefficient of restitution is being taken as 1 (fully elastic) in these particular analyses. It is clearly observed from the graphs of surge, sway, roll and yaw amplitudes that with the increase in collision time the respective amplitudes are reducing very significantly. This implies the importance of collision contact period and also of the restitution period of the fender materials which are needed to have larger duration in order to produce lesser amplitudes during collisions between different vessels.

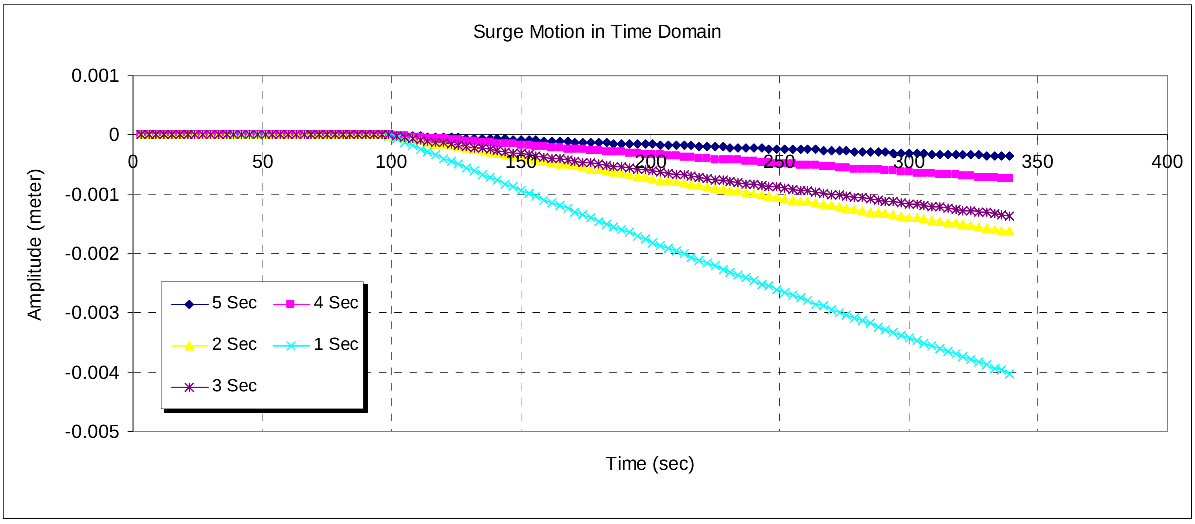


Figure 4.95: Surge motion for various collision times (struck at 1 knot).

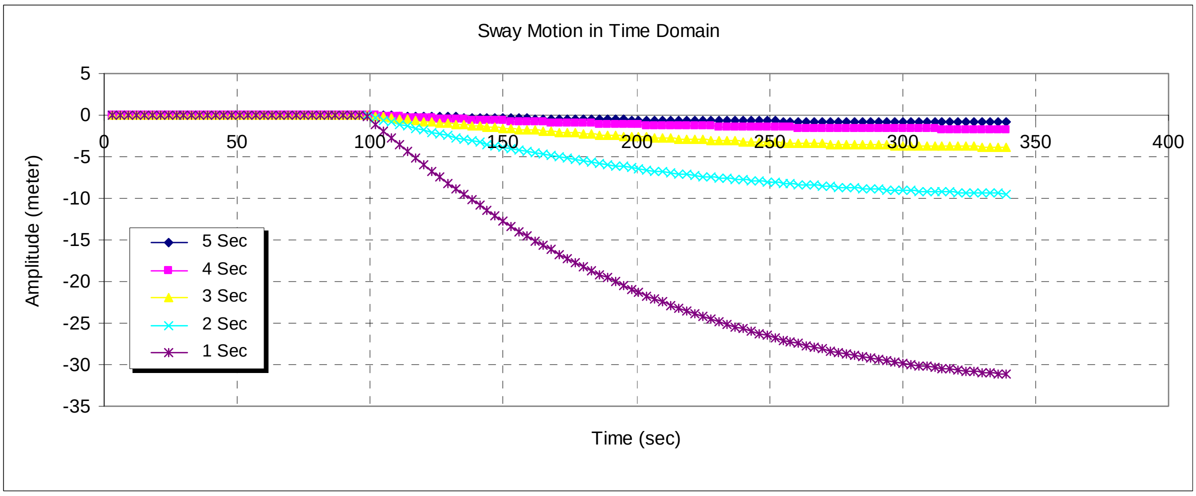


Figure 4.96: Sway motion for various collision times (struck at 1 knot).

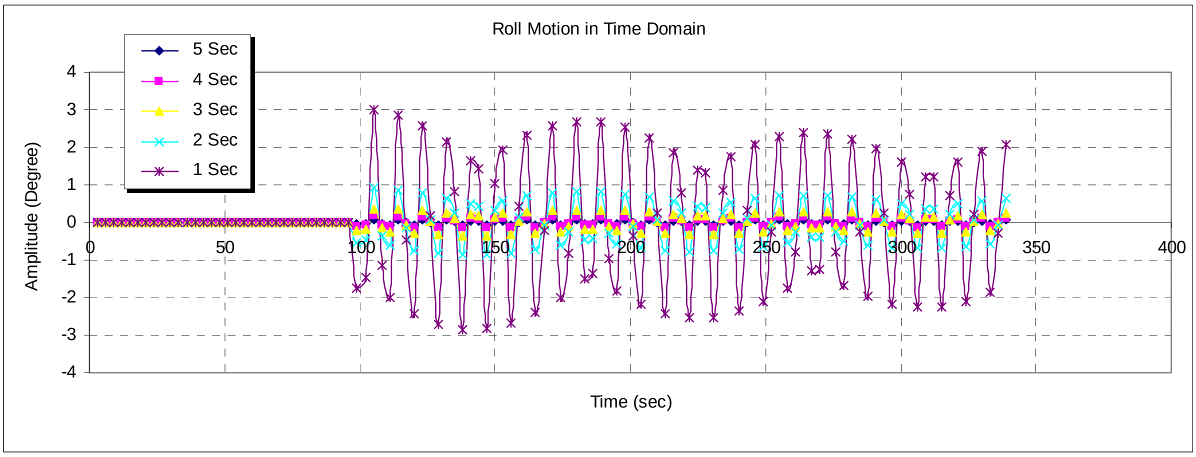


Figure 4.97: Roll motion for various collision times (struck at 1 knot).

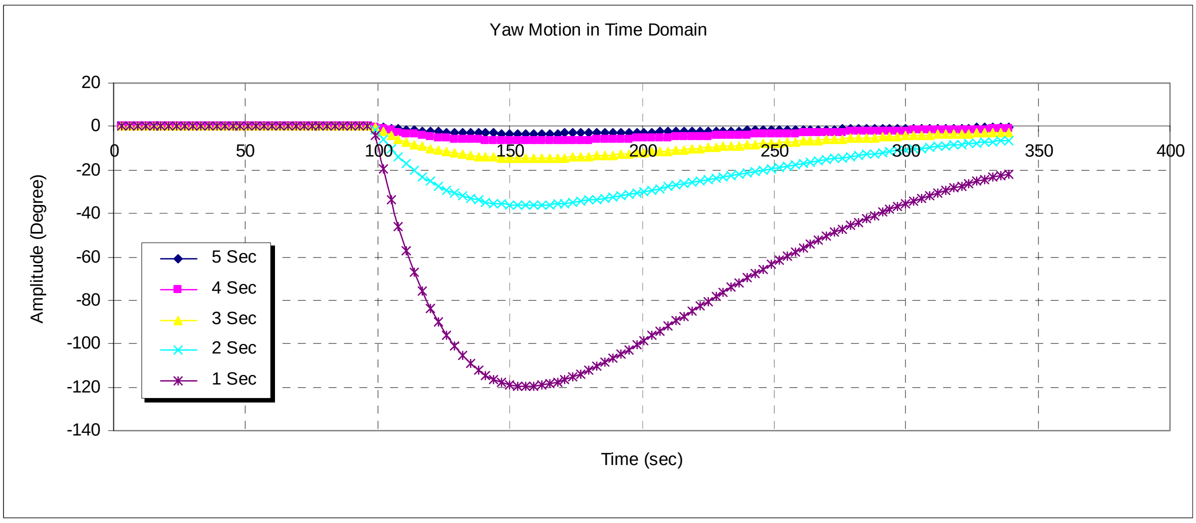


Figure 4.98: Yaw motion for various collision times (struck at 1 knot).

Similar observations are being made in cases 13 to 16, 19 and 22 where the rolling amplitudes, for coefficient of restitution one, exceeds the struck ships angle of vanishing stability. This feature is important to understand because of the fact that these cases represent catastrophic capsizing scenarios. The struck ships in these cases are being rolled excessively due to the two main reasons applicable in these particular cases, which are i) Higher coefficient of restitution (E = 1.0) and ii) Lower collision time (1 sec). Therefore, further analysis is being conducted on case 22 (32m vs. 46m at L/4 aft of midship at 90 ° at speed 6 knot) by altering the collision contact period as shown in the Figures 4.99, 4.100, 4.101 and 4.102. It has been observed from the analysis that the surge, sway, roll and yaw motions are being reduced very significantly with the increase in collision contact period.

However, the most important observation is that the roll amplitude of the struck ship is reduced very significantly around 48 degree from over 130 degree only by a change in collision contact period of 1 second as seen in Figure 4.101. This resembles a critical scenario where a ship becomes vulnerable to capsizing only due to materials of higher restitution period that are being used in the fenders. Thus this risk of capsizing can be eliminated by applying materials that take more time to restitute back after collision or compression to be more specific.

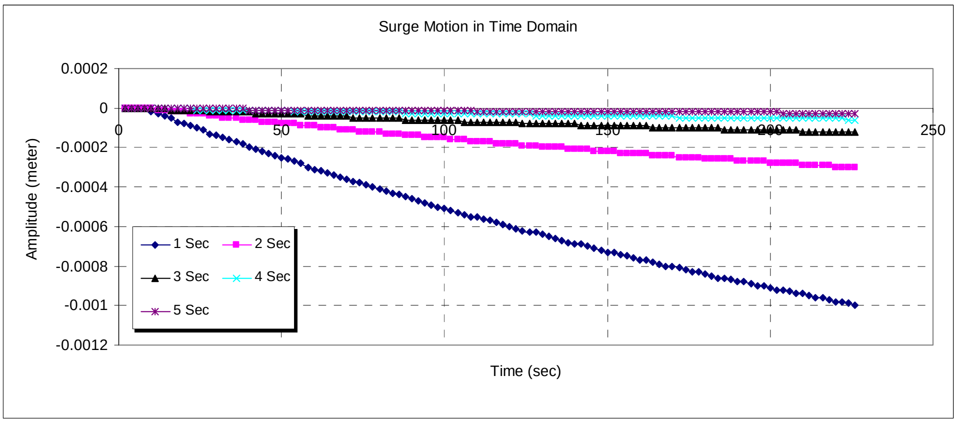


Figure 4.99: Surge motion for various collision times (struck at 6 knot).

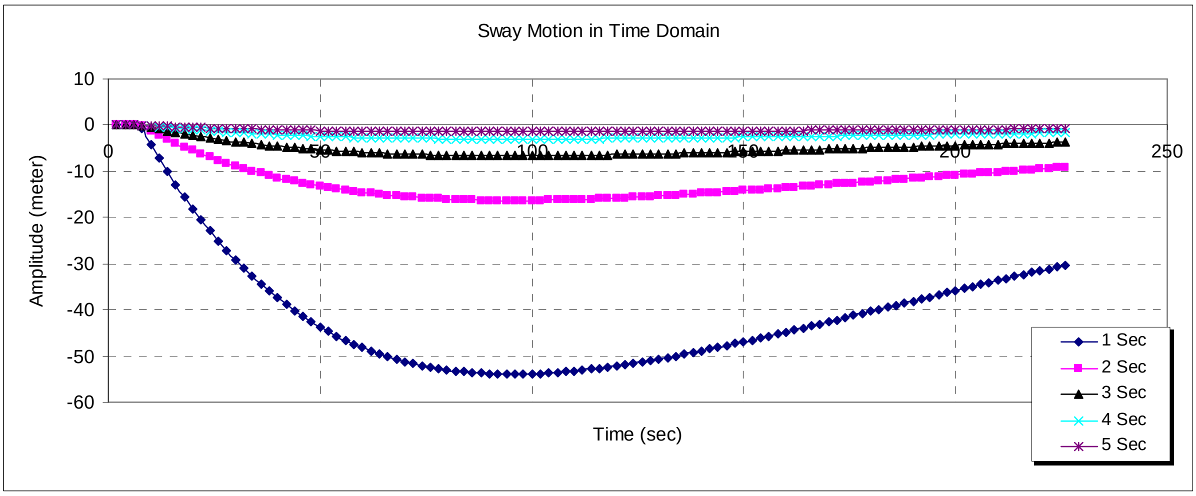


Figure 4.100: Sway motion for various collision times (struck at 6 knot).

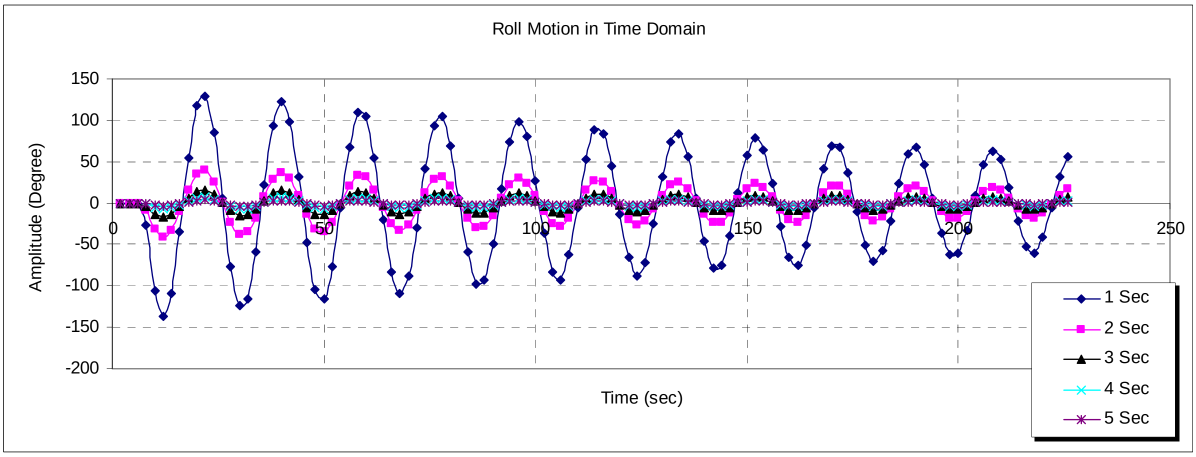


Figure 4.101: Roll motion for various collision times (struck at 6 knot).

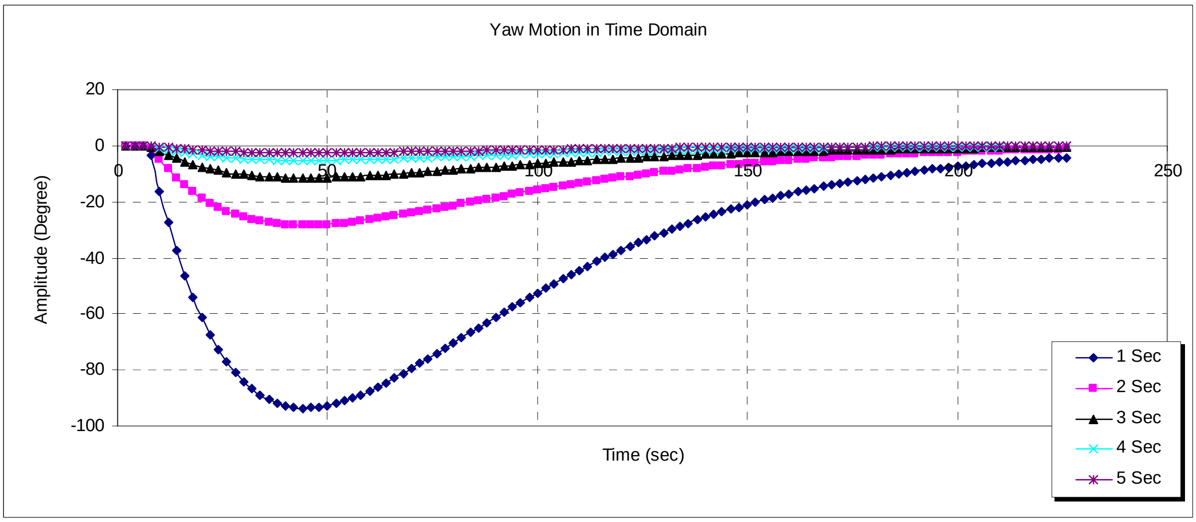


Figure 4.102: Yaw motion for various collision times (struck at 6 knot).

4.5 Summary of the Analyses

A set of numerical computations have been carried out to investigate the various aspects of dynamic behaviour of ships at collision. The results in detail are presented in the previous sections. However, the outcomes of the investigation draw some important findings, the summary of which are as follows:

- It is observed from the analysis of forces against various striking ship's speed that the collision force on the struck ship is linearly proportional to the striking ship's speed.

- It is also noticed that for a particular speed of attack the force exerted on the struck ship varies significantly with the changes in various coefficient of restitutions. The variation in force due to the coefficient of restitution is significant at higher speeds and becomes less noticeable at lower speeds; suggesting that the use of lower restitution material are more effective in damping out the collision force at higher speeds.

- The collision force reduces drastically with the increase in collision time or contact period and vice versa. The change is significant in between time 0 (zero) and 1 second. But the collision force reduces very insignificantly as collision time increases after 1 second. The collision force approaches to zero as collision time increases while the force approaches infinity as collision time tends to zero.

- Added mass for force in the sway direction plays a significant role as well. It is observed that higher the added mass the lower is the collision force and quite oppositely, the lower is the added mass the higher is the collision force in sway direction.

- The collision force is maximum at the direction along the striking ships velocity vector and is minimum at the direction perpendicular to the velocity vector of the striking ship.

- Fully plastic collision (coefficient of restitution, $E = 0$) can reduce the surge, sway, roll, pitch and yaw amplitudes by 60 to 85 percent in comparison to fully elastic collision (i.e. when $E=1.0$).

4.6 The Application and Results of Collision Avoidance Chart (CAC)

This study develops a collision avoidance chart that can be used to calculate the arrival and departure time of the ships at the collision zone and thus may try to avoid effectively a possible collision by altering the course and/or ship speed. The following two examples show how collision could be avoided by altering relative speeds. It will be seen from the charts that only by adopting an alteration in the speed of Ship A, where the observer is standing and calculating the charts, an inevitable collision can be avoided.

Case 1: Ship A Strikes Ship B

Straight line distance between ships, bow to bow, r = 100 m

Angle with the direction of heading of Ship A, θ = 60 degree

Speed of Ship A, Va = 2 m/s

Speed of Ship B, Vb = 2 m/s

Length of Ship A, La = 30 m

Length of Ship B, Lb = 10 m

Heading of Ship A = 090

Heading of Ship B = 135

In this first case, from the detail particulars given above, it is observed that two ships are plying in such a course that after some time they are about to collide with each other at the collision zone which is shown in the Collision Avoidance Chart (CAC) Case 1 as the point (Xc, Yc). Applying a series of calculations, the arrival and departure time at the collision zone of both of the ships are being calculated. It is seen that Ship B arrives at the collision zone at 61st second and about to leave the point on 85th second. However, unfortunately Ship A at its current speed of 2 m/sec arrives at that point on 68th second which is before Ship B leaves the collision point. Therefore, there is an inevitable collision. This phenomena is more clearly understandable as the time lines are being drawn at the CAC Case 1. In the next case it will be seen that by simply altering the speed of Ship A the collision could be avoided.

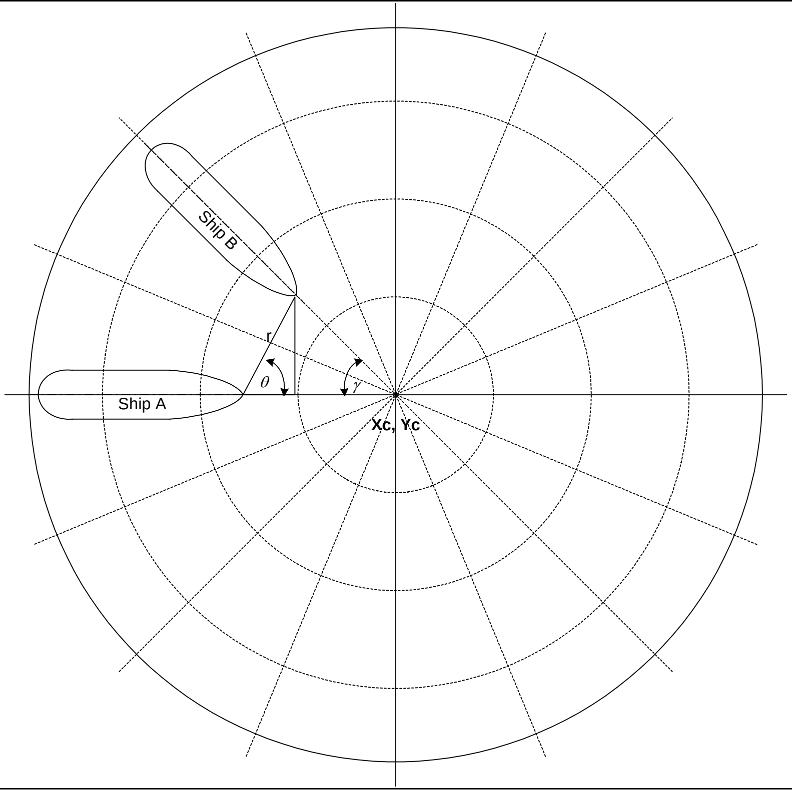
Case 2: No Collision

All the particulars remain the same except the speed of Ship A is reduced.

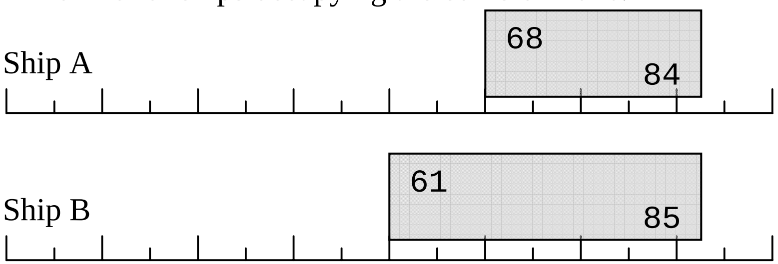
Speed of Ship A, Va = 1.5 m/s

In this case the speed of Ship A has been reduced down to 1.5 m/s. This is because Ship B appears to be arriving early at the collision point; therefore, it will be a wise decision to reduce the speed of Ship A and allow Ship B to pass through the point at first. As it is seen in CAC case 2, the time line suggests that Ship A arrives much late due to a reduction in speed at the 91st second which is indeed much later than Ship B leaves the collision point at the 85th second. It is thus been demonstrated that using this simple chart a major catastrophe could be prevented by adopting only a simple measure of reducing the speed.

Collision Avoidance Chart (CAC) for Case 1



Time line for ships occupying the collision zone:



DISTANCE, R = **100** ANGLE, θ = **60**

SPEED OF SHIP A, VA = **2** SPEED OF SHIP B, VB = **2**

LENGTH OF SHIP A, LA= **30** LENGTH OF SHIP B, LB= **46**

HEADING OF SHIP A = **90** HEADING OF SHIP B = **135**

γ = HEADING OF B - HEADING OF A = 135-90 = **45**

$x_{Bbow} = r \times \cos\theta$ = 100 x 0.5 = **50**

$y_{Bbow} = r \times \sin\theta$ = 100 x 0.87 = **87**

$x_{Bstern} = x_{Bbow} - L_B \times \cos\gamma$ = 50 – 46 x 0.71 = **17.34**

$y_{Bstern} = y_{Bbow} + L_B \times \sin\gamma$ = 87 + 46 x 0.71 = **119.66**

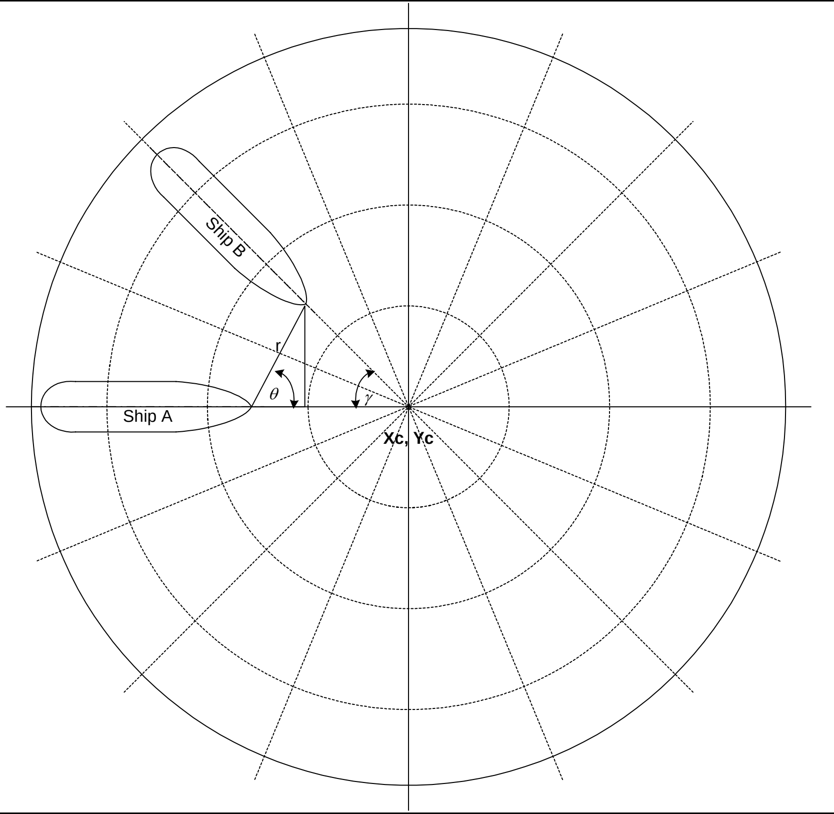
$$X_C = x_{Bbow} - \frac{y_{Bbow} \times (x_{Bbow} - x_{Bstern})}{(y_{Bbow} - y_{Bstern})}$$ =50 + 87 x 32.66 / 32.66

= **137.0**

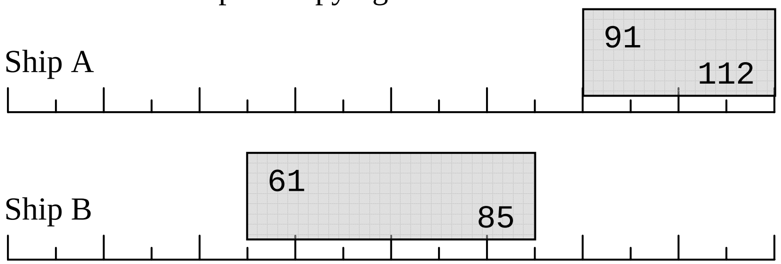
$TA_{arrival} = \frac{X_c}{V_A}$ = **68** $TA_{departure} = \frac{X_c + L_A}{V_A}$ = **84**

$TB_{arrival} = \frac{X_c - x_{Bbow}}{V_B \cos\gamma}$ = **61** $TB_{departure} = \frac{X_c - x_{Bstern}}{V_B \cos\gamma}$ = **85**

Collision Avoidance Chart (CAC) for Case 2



Time line for ships occupying the collision zone:



DISTANCE, R = **100** ANGLE, θ = **60**

SPEED OF SHIP A, VA = **1.5** SPEED OF SHIP B, VB = **2**

LENGTH OF SHIP A, LA= **30** LENGTH OF SHIP B, LB= **46**

HEADING OF SHIP A = **90** HEADING OF SHIP B = **135**

$\gamma =$ HEADING OF B - HEADING OF A = 135-90 = **45**

$x_{Bbow} = r \times \cos\ \theta$ = 100 x 0.5 = **50**

$y_{Bbow} = r \times \sin\ \theta$ = 100 x 0.87 = **87**

$x_{Bstern} = x_{Bbow} - L_B \times \cos\gamma$ = 50 – 46 x 0.71 = **17.34**

$y_{Bstern} = y_{Bbow} + L_B \times \sin\gamma$ = 87 + 46 x 0.71 = **119.66**

$$X_C = x_{Bbow} - \frac{y_{Bbow} \times (x_{Bbow} - x_{Bstern})}{(y_{Bbow} - y_{Bstern})}$$ =50 + 87 x 32.66 / 32.66

= **137.0**

$TA_{arrival} = \dfrac{X_c}{V_A}$ = **91** $TA_{departure} = \dfrac{X_c + L_A}{V_A}$ = **112**

$TB_{arrival} = \dfrac{X_c - x_{Bbow}}{V_B \cos\gamma}$ = **61** $TB_{departure} = \dfrac{X_c - x_{Bstern}}{V_B \cos\gamma}$ = **85**

Chapter 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Concluding Remarks

The research on studying the dynamic behaviour of ships for different coefficient of restitutions of the hull material is indeed a new concept and so far limited knowledge is available to the researchers in this particular area. Therefore, as a preliminary investigation the study is limited to mathematical formulations only and due to deficiency in infrastructural facilities for experimental investigations the study had no other options but to validate its results with published numerical results.

5.2 Recommendations

Based on the knowledge obtained from this particular study the following recommendations can be drawn:

- The materials used in the construction of fenders and other external protective devices or structural members are recommended to have as much lower coefficient of restitution as possible. Such adaptation significantly reduces the collision forces imparted on the dynamics of ships in events of high speed collision.

- Due to practical limitations, if materials with lower restitutions do not get the economical feasibility, the compensation may be made by using materials that have longer duration for restitution. This will reduce the collision force significantly although it may restitute hundred percent after being compressed due to external striking force but the change in momentum will be decreased due to larger value of time which will result reduced collision force on the ship. This type of fender may be economically feasible for using in ships over a longer period.

- The use of Collision Avoidance Chart (CAC) may be suggested which appears to be an efficient and cost effective solution in preventing collisions. However, the mathematics developed in the chart may be incorporated in to a computer as a software and can be used more efficiently by adding visual features. Although incorporating a computer on every vessel may not be possible in Bangladesh due to economical limitations, a smaller and cheaper device may be designed using the developed algorithm and be marketed at a reasonable price. Further research on it is indeed recommended.

- Under the prevailing situation, it might be a cost effective measure to impose restrictions on some vessels in plying highly congested water areas, particularly boats those are unable to withstand collision. It is observed that the smaller boats in Bangladesh are mostly used for shorter trips and carry excessive passengers (e.g. Kheya Boats) and when these vessels encounter an accident they cause a high number of fatalities. Therefore, new legislations may be activated on collision worthiness particularly giving emphasis on smaller boats and cargo ships as well.

- It is observed that there is a much more scope of research in this particular are of collision dynamics. Future study may be conducted on smaller country boats and with more detailed description of the waterways as the river width, depth, geometry etc which may play a notable role in such investigations. Further study with wave and wind consideration is also recommended as it may influence the ships dynamics either adversely or favourably during a collision. Experimental investigations are indeed recommended for future study.

References

1. Banglapedia, Water Transport, National Encyclopaedia of Bangladesh, Web Page Address:http://www.banglapedia.net/HT/W_0034.HTM, 2005.

2. Transport & Communications, Bangladesh Economic Review (2004), Chapter 11, pp. 127-158, 2004.

3. Report of the Task Forces on Bangladesh Development Strategies for the 1990's, Volume – IV, *University Press Limited*, Dhaka, Bangladesh, 1991.

4. Islam, M.R., Techno-economic Analysis Applied to Safety of Inland Passenger Vessels, *M.Sc. Engg. Thesis, Department of Naval Architecture and Marine Engineering, Bangladesh University of Engineering & Technology (BUET)*, 1997.

5. Zahanyar, S.I.M. and Haque, M.E., Investigation of Accidents, Damages and Cargo Losses in Inland Shipping, *B.Sc. Engineering Project Thesis, Department of Naval Architecture and Marine Engineering. Bangladesh University of Engineering & Technology (BUET)*, Dhaka, Bangladesh, 1989.

6. Bangladesh Transport Sector Study (BTSS), Volume I, *Planning Commission of Bangladesh, Dhaka*, Bangladesh, 1994.

7. Safe, Efficient and Improved Water Transport System: Problems and Prospects, Seminar Report, *Bangladesh Inland Water Transport Authority (BIWTA)*, Dhaka, Bangladesh, 2003.

8. Khalil, G.M. and Tarafder, M.S., Ferry Disasters in The Rivers of Bangladesh: Some Remedial Measures, *The Journal of National Oceanographic And Maritime Institution*, Vol. 21, No. 2, pp. 33-53, December 2004.

9. Chowdhury, A.S., Waterway Accident Characteristics Assessment and Information System Development, *Master of Engineering (Civil and Transportation) Thesis, Department of Civil Engineering, Bangladesh University of Engineering & Technology (BUET)*, Dhaka, Bangladesh, 2005.

10. Awal, Z.I., A Study on Inland Water Transport Accidents in Bangladesh: Experience of A Decade (1995-2005), *Proceedings of the International Conference on Coastal Ships & Inland Waterways*, pp. 67-72, Royal Institution of Naval architects (RINA), London, UK, 15-16 March 2006.

11. Awal, Z.I., Islam, M.R. and Hoque, M.M.H., An Analysis of Passenger Vessel Accidents in the Inland Waterways of Bangladesh", *Proceedings of the Marine Technology Conference 2006*, pp. 211-219, University of Hasanuddin, Indonesia, 4-5 September, 2006.

12. Awal, Z.I., Islam, M.R. and Hoque, M.M.H., A Study on Inland Water Transport Accidents: Some Causes and Recommendations, Presented as the Key Note Paper in a Round Table Meeting on "Assono Jhoro

Mousume Nou Poribohon Malik O Amader Karonio", National Press Club, Dhaka, 11 March, 2006.

13. Minorsky, V.U., An Analysis of Ship Collisions With Reference to Protection of Nuclear Power Plants, *Journal of Ship Research*, Vol 3 No. 1, pp. 1-4, October 1959.

14. Crake, K.W., Probabilistic Evaluation of Tankship Damage in Collision Events, *Naval Engineer & Master of Engineering in Ocean Engineering Thesis, Massachusetts Institute of Technology (MIT) and the Woods Hole Oceanographic Institute*, USA, June 1998.

15. Zhang, S., The Mechanics of Ship Collisions, *Ph.D. Thesis, ISBN 87-89502-05-1, Department of Naval Architecture & Offshore Engineering, Technical University of Denmark*, DK-2800 Lyngby, Denmark, January 1999.

16. Woisin, G., "Design Against Collision", *Schiff & Hafen*, Vol. 31, No. 2, pp. 1059-1069, Germany, 1979.

17. Amdahl, J., Energy Absorption in Ship-Platform Impact, *Norwegian Institute of Technology*, Report No. UR-83-34, 1983.

18. Jones, N., A Literature Survey on the Collision and Grounding Protection of Ships, *Ship Structures Committee Report*, SSC-283, 1979.

19. Ellinas, C.P. and Valsgard S., Collision and Damage of Offshore Structures a State of the Art, *4th Int. Symposium on Offshore Mechanics and Arctic Engineering*, Dallas, Texas, February 17-22, 1985.

20. Samuelides, E and Frieze P.A., “Fluid-structure Interaction in Ship Collisions”, *Marine Structures*, Vol. 2, pp. 65-88, 1989.

21. Pedersan, P.T., Valsgaard, S., Olsen, D., and Spangenberg, S., Ship Impacts: Bow Collisions, *International Journal of Impact Engineering*, Vol. 13, No.2, pp. 163-187, 1993.

22. Qvist, S., Nielsen, K.B., Schmidt, M.H. and Madsen S.H., Ship Collision – Experimental and Numerical Analysis of Double Hull Models, *9th DYMAT Technical Conference*, 1995.

23. Dimitris P. Servis and Manolis Samuelides, Ship Collision Analysis Using Finite Elements, *SAFER EURORO Spring Meeting*, Nantes 28 April 1999.

24. Brown, A.J. and Chen, D., Probabilistic Method for Predicting Ship Collision Damage, *Ocean Engineering International Journal*, Vol. 6 No. 1, pp. 54-65, 2002.

25. Brown, A.J., Tikka, K., Daidola, J., Lutzen, M. and Choe, I., Structural Design and Response in Collision and Grounding, *SNAME Transactions*, Vol. 108, pp. 447-473, 2000.

26. Chen, D., Simplified Ship Collision Model, *Master of Science in Ocean Engineering Thesis, Virginia Polytechnic Institute and State University, Blacksburg, Virginia*, USA, January 2000.

27. Anon Rosenblatt & Son, Inc, Tanker Structural Analysis for Minor Collision, *USCG Report*, CG-D-72-76, 1975.

28. McDermott, J.F., Tanker Structural Analysis for Minor Collisions, *SNAME Transactions*, Vol. 82, pp. 382-414, 1974.

29. Reardon, P. and Sprung, J.L., Validation of Minorsky's Ship Collision Model and Use of the Model to Estimate the Probability of Damaging a Radioactive Material Transportation Cask During a Ship Collision, *Proceedings of the International Conference on Design and Methodologies for Collision and Grounding Protection of Ships*, San Francisco, August 1996.

30. W. Abramowicz, and M. Sinmao, User's Manual and Modeling Guide for the Program DAMAGE, *Joint MIT-Industry Program on Tanker Safety*, Report No. 66, June 1999.

31. Salvesen, N., Tuck , E.O. and Faltinsen, O., Ship Motions and Sea Loads, *SNAME*, Vol 78, pp. 250-287, 1970.

32. Vugts, J.H., The Hydrodynamic Forces and Ship Motions in Oblique Waves, Research Centre, *TNO for Ship Building*, Delft Report No. 1505, 1971.

33. Bhattacharyya, R., Dynamics of Marine Vehicles. *John Wiley & Sons, New York,* 1978.

34. Faltinsen, O.M. and Michelsen, F.C., Motions of Large Structures in Waves at zero Froude number, *International Symposium on the*

Dynamics of Marine Vehicles and Structures in Waves, Mechanical Engineering, London, pp. 99-114. 1975.

35. Hogben, N. and Standing R.G., Wave loads on large bodies, *International Symposium on the Dynamics of Marine Vehicles and Structures in Waves, Mechanical Engineering*, London, pp. 273-292. 1975.

36. Garrison, C.J., Hydrodynamic loading of large offshore structures: Three dimensional source distribution methods, *Numerical Methods in Offshore Engineering*, pp. 87-140, 1974.

37. Inglis, R.B. and Price W.G., Calculation of the velocity potential of a translating, pulsating source, *Royal Institution of Naval Architects (RINA) Transactions*, 1980.

38. Inglis, R.B. and Price W.G., A three dimensional ship motion theory: Calculation of wave loading and responses with forward speed, *Royal Institution of Naval Architects (RINA) Transactions*, 1981.

39. Islam, M.N., Islam, M.R., Baree, M.S., Computation of Ship Responses in Waves Using Panel Methods, *Journal of Naval Architecture and Marine Engineering*, Vol 1, No. 1, pp. 35-46, 2004.

40. Oztas, A.G., Application of Impulse Momentum Theory to Vehicle Collisions, *Turkish Journal of Engineering & Environmental Science*, Vol 23, pp. 455-464, 1999.

41. Petersen, M.J., Dynamics of Ship Collision, *Ocean Engineering*, Vol. 9, No. 4, pp. 295-329, 1982.

42. Hanhirova, H., External Collision Model, Safety of Passenger/RoRo Vessels, *Helsinki University of Technology, Ship Laboratory*, October, 1995.

43. Motora S., "Equivalent Added Mass of Ships in Collisions", Selected Papers from Society of Naval Architects, Japan, Vol. 7, pp. 138-128, 1971.

44. Pedersen, P.T., Valsgaard, S., Olsen, D. and Spangenberg S., Ship Impacts: Bow Collisions, *International Journal of Impact Engineering*, Vol. 13, No. 2, pp. 163-187, 1993.

45. Hassan, M.M., Motion Simulation of a Naval Vessel with Forward Speed in a Rough Sea, M.Sc. Thesis, Department of Naval Architecture & Marine Engineering, *Bangladesh University of Engineering & Technology (BUET)*, Dhaka, 2003.