Development of a Mathematical Model for Analysis on Ship Collision Dynamics

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Abstract :- This research work attempts to develop a model for ship to ship collision. The collision model is fundamentally divided into two segments namely, model for simulation before collision and model for simulation after collision. In the first part, mathematical formulations are derived for finding the possibility of a collision, determining the spatial location of collision and identification of the contact points on the ships. In the later part, a mathematical model is developed to study the kinetic energy losses, collision forces and dynamic responses with respect to different variables such as coefficient of restitution, ship speed, angle of attack, location of hitting, added mass for sway force and others. In the model, expressions for collision forces are derived based on changes in linear momentum. By incorporating the collision force into the equation of motion, which is a linear differential equation with constant coefficients, the dynamic responses are calculated for different collision scenarios. The study considered two different vessels of length 46 meter and 32 meter for conducting the simulations. Results obtained from the mathematical model suggest that collision forces can be reduced significantly by altering the considered variables; e.g. motion amplitudes can be reduced very significantly (as high as eighty five percent) by using materials with lower coefficient of restitution in the fenders and may save ships from capsizing in severe cases. Finally, a number of recommendations have been put forward and further investigations on such models are also proposed.

Key Words:- Collision Dynamics, Coefficient of Restitution, Collision Forces, Ship Capsizing, Mathematical Model

1. Introduction

The inland shipping like many other riverine countries of the world plays a very significant role in Bangladesh transporting passengers and goods within the country as the waterways are both extensive and well connected with the other modes of transportation such as roadways and railways. In terms of traffic intensity, the inland waterway network generates about 1.57 million passenger-kilometres per route-kilometre of waterway. The density of inland ports and terminals is much higher on the inland waterways with approximately 3.7 berthing facilities per 100 route-kilometres. The density of passenger facilities on the inland waterways is also high at around 40 per 100 route-km [1].

With the increase in population and the growing economy of the country as well the waterways are getting highly congested and problems relating to maritime safety are emerging at a totally different scale. In Bangladesh maritime safety has become a severe issue in quick succession when a number of passenger launches capsized killing several thousands of people within the past few years. The underlying causes of these accidents were mainly the seasonal storms (known as Nor'wester) and collision between ships. In response to such emergencies, the government took some serious actions of strict embargo for ships plying in inclement weather. This, however, has provided some significant improvement to the safety scenario despite the fact that collision between marine vessels are still taking place and there appears the need for some serious in depth investigations. Studies by Awal et. al [2] reveal that 56 percent of all the passenger launch accidents are collision due to human error and only 21 percent being the loss of stability due to Nor'wester. Studies on overall accident characteristics [3,4] also suggest that collisions between marine vessels are also significantly higher in comparison to other types of accidents.

The fact that concerns all is that the collision accidents are fatal and the extent of damage and loss of property are tremendously expensive which puts considerable burden on the national economy. There remain numerous deficiencies on maritime safety and the scope for improvements in this area is a contemporary demand. This study is therefore, an attempt to improve safety of ships by studying the possibility of a collision and thus inventing ways to prevent it cost effectively and efficiently. Also most importantly, this paper attempts to develop a mathematical model to study the dynamic characteristics of the vessels during a collision considering speed, angle of attack, coefficient of restitution of the fender material and virtual mass of the ships as variables. For simplicity the study excludes the wave effects and considers uncoupled rolling motion of the struck ship since the motion is directly related to the capsizing.

2 Literature Review

The risks involved and the consequences associated with ship-ship collisions are extremely high and catastrophic. Particularly the environmental and economical issues create a huge impact in the community when these catastrophic incidents take place. One of the early pioneers to recognise such problems and to conduct mathematical analyses based on empirical models was Minorsky [5]. In October 1959, Minorsky published a research paper where he analysed ship collision with reference to protection of nuclear power plant. Minorsky followed a semi-analytical approach based on the facts of actual collision. The objective of his work was to predict with some degree of accuracy the conditions under which a nuclear ship remain intact and, consequently, what structural strength should be built into the reactor space within the hull in order to absorb safely a given amount of kinetic energy in a collision. Minorsky's method consisted of relating the energy dissipated in a collision event to the volume of damaged structure. In the original analysis the collision is assumed to be totally inelastic, and motion is limited to a single degree of freedom.

Zhang [6] in his doctoral research work developed models for ship collisions where collision energy loses, collision forces and structural damages were determined. The analysis procedures were divided into two parts: the external dynamics and the internal mechanics. By combining the outer analysis and the inner analysis, a number of examples for full-scale ship collisions were analysed and finally a method relating the absorbed energy and the destroyed material volume was developed and verified. His approach overcome a major drawback of Minorsky's well known method since it takes into account the structural arrangement, the material properties and damage modes.

During the past fifty years a number of model experiments have been carried out in Italy, Germany and Japan. The principal objectives of these tests were to design nuclear powered ships having adequate protection to the nuclear reactor from collision damage. Several authors have given detailed reviews on these experiments, for example, Woisin [7], Amdahl [8], Jones [9], Ellina and Valsgard [10], Samuaelides [11] and Pedersen et al. [12]. During the period of 1967 to 1976, 12 model ship collision tests were carried out in Germany (Woisin, 1979). The model scales range from 1/12 to 1/7.5 where the test setups resembled a striking bow running down from an inclined railway path hitting the side shell of a ship.

In some recent investigations, powerful computers are used to model collision scenarios using finite elements. Simulation programs started to run into the computers and events could be seen in time frame, second by second. According to Dimitris et al. [13] there are two major questions that naval engineers working on ship collisions should approach: One concerns the simulation of ship collisions and the prediction of the damages, which occur during the incident. The other is the identification of collision scenario or scenarios, which the ship under consideration should be checked against in order to assess her capacity to withstand collision loads. Dimitris in his work used extensive finite element codes for collision simulation.

Brown and Chen [14], Brown et.al [15] and Chen [16] conducted extensive work in developing Simplified Collision Model (SIMCOL) based on solutions of external dynamics and internal deformation mechanics in time domain simulations for the rapid prediction of collision damage in probabilistic analysis. The external sub-model used a three-degree of freedom system for ship dynamics. The internal sub-model determined reacting forces from side and bulkhead structures using mechanisms adapted from Rosenblatt and McDermott [17,18], and absorbed energy by decks, bottoms and stringers calculated using the Minorsky's correlation as modified by Reardon and Sprung [19].

A computer program DAMAGE was developed at the Massachusetts Institute of Technology (MIT) under the Joint MIT-Industry Program on Tanker Safety [20]. The joint intervention produced several versions of the program and the program

DAMAGE V5.0 was claimed to have considerable success in predicting structural damage in the accident scenarios such as ship grounding on a conical rock (rigid rock, deferrable bottom) and ship-ship collision (deferrable side, deferrable bow). A major advantage of DAMAGE, as proclaimed by the authors, is that the theoretical models are hidden behind a modern graphical user interface (GUI).

Interestingly all these research works were intended to investigate the structural performances during collision with the objectives of providing watertight integrity, safeguarding the extremely valuable passenger, cargo, and other important resources. Indeed there have been considerable advances in developing methodologies and formulations of determining the collision damages. However, none of the research works have looked upon the dynamic characteristics of the ships during a collision event, particularly the aspects of stability with reference to capsizing due to excessive rolling by collision force. Therefore, it is the purpose of this research is to develop a ship collision dynamics model and investigate the phenomenon under potentially dangerous circumstances.

3 Development of Theoretical Model

The mathematical model developed in this study can be divided into two fundamental segments with reference to the time domain analysis; such as: (1) Before collision model and (2) During and after collision model. However, for both of the segments there are some fundamental assumptions adopted and these are as follows:

1. It is assumed, only while determining the hitting position at the struck ship abaft the bow, that the ships are straight line objects and their breadths and curved body shapes are ignored.
2. There is no friction or sliding between the striking and struck ship. The ships get separated from each other after the collision.
3. It is also assumed that the ships do not encounter a second collision after being hit at the first instance so that the ships can have free motions in space after the incident.
4. For time domain simulation the collision time (contact period between the two ships) is assumed as one second. However, for additional analyses the contact period has been considered as a variable in between 1 to 5 seconds.
5. It is also assumed while dealing with the equation of motion that there are no waves or wind forces before and after collision.

3.1 Model for Simulation before Collision

Initially it is considered that two ships are plying in waters each having their own particular forward speeds and headings. It is, at this stage, considered fundamental to find out the four arise four questions that are considered fundamental for the mathematical formulations:

1. With the given speeds and headings, will there be a collision between two ships?
2. If there is a collision, where will it take place (i.e the location)?
3. Which ship hits the other?
4. Which part of the ship along its length the hitting takes place?

In order to find the answers of these questions it is crucial to locate the point of intersection of the paths of the vessels. If it is assumed that the given vessels keep their course unaltered, there will be a certain point at which their straight-line paths of will intersect each other. This point may be defined as the Point of Intersection of the Paths. This is, however, not necessarily be the point at which the ships get struck rather it gives an idea about the location certain distance away from the ships where the accident will take place.

Since it is assumed that the ships remain in their heading and do not alter their course, their paths can therefore, be expressed as straight lines. Considering Fig. 1 let the paths of the vessels be:

$$y = m_A x + c_A \qquad \text{(For Ship A)}$$

$$y = m_B x + c_B \qquad \text{(For Ship B)}$$

Where, m_A & c_A and m_B & c_B are the slopes and constants of Ship A and B respectively which depend on the ships heading and relative position on the co-ordinate system.

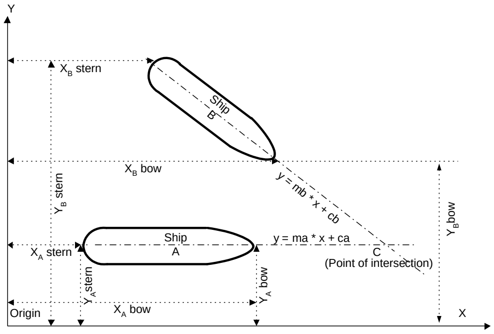


Fig. 1: Co-ordinate system for simulation before collision.

An origin is assumed at a suitable place and the slopes of these straight lines are obtained from the relative position of their sterns and bows in Cartesian co-ordinates.

Let the point of intersection be defined by C (X_c, Y_c) and thereby it is obtained as,

$$X_C = -\left(\frac{c_A - c_B}{m_A - m_B} \right)$$

$$Y_C = \frac{m_B c_A - m_A c_B}{m_A - m_B}$$

Knowing the point of intersection gives the probable idea of the location of collision. However, the incident of collision can still be avoided by altering the speeds of the vessels by keeping the heading unaltered. Therefore, it is important to know whether both the ships occupy the point of intersection at the same time or not. If they occupy the point at the same time, the collision is inevitable. On the other hand, if any of the ship passes through before arrival of the other or arrives late while allowing the other to pass through, the collision could be avoided. This critical situation may well be similar to when vessels lose control of their rudder on a collision course and have only the freedom of varying their respective speeds.

Let the measured parameters of relative positions of the ships with respect to origin were taken at time t = 0. If one of the ship's bow (say Ship A) arrives at the intersection point at time $TA_{arrival}$ and the stern leaves the point at time $TA_{departure}$, then the occupation duration is the difference between $TA_{departure}$ and $TA_{arrival}$. In such a case if a collision is to take place, the other ship must arrive the point in between time $TA_{departure}$ and $TA_{arrival}$. Therefore, it is essential to know the arrival and departure time of both of the ships at the point of intersection or may be defined as the "Collision Zone". An example is shown in Fig. 2 where the scenario is depicted. In the first case both of the ships occupy the collision zone at the same time as seen by the overlapping time line of the ships. However, in the second case the ships occupy the collision zone at different time intervals and avoid a potentially dangerous collision.

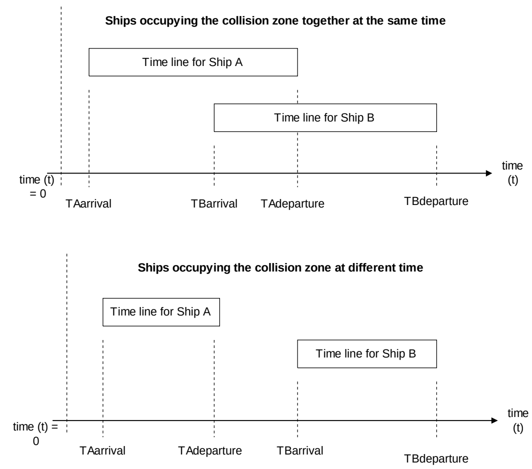


Fig. 2: Time line description of the ships occupying the collision zone.

The arrival time and departure time of the ships ($TA_{arrival}$, $TB_{arrival}$, $TA_{departure}$, $TB_{departure}$) are obtained as the following,

$$TA_{arrival} = \frac{X_c - x_{Abow}}{V_A \cos m_A}$$

$$TA_{departure} = \frac{X_c - x_{Astern}}{V_A \cos m_A}$$

Similarly for ship B the following are obtained:

$$TB_{arrival} = \frac{X_c - x_{Bbow}}{V_B \cos m_B}$$

$$TB_{departure} = \frac{X_c - x_{Bstern}}{V_B \cos m_B}$$

If Ship A arrives at the point of intersection before Ship B and Ship A doesn't depart before arrival of Ship B (i.e. $TA_{arrival} < TB_{arrival} < TA_{departure}$), in such a case Ship B hits Ship A and the collision occurs at $TB_{arrival}$. On the other hand, if Ship B arrives at the point of intersection before Ship A and Ship B doesn't depart before arrival of Ship A (i.e. $TB_{arrival} < TA_{arrival} < TB_{departure}$), in such a case Ship A hits Ship B and the collision occurs at $TA_{arrival}$. Fig. 3 depicts the two situations.

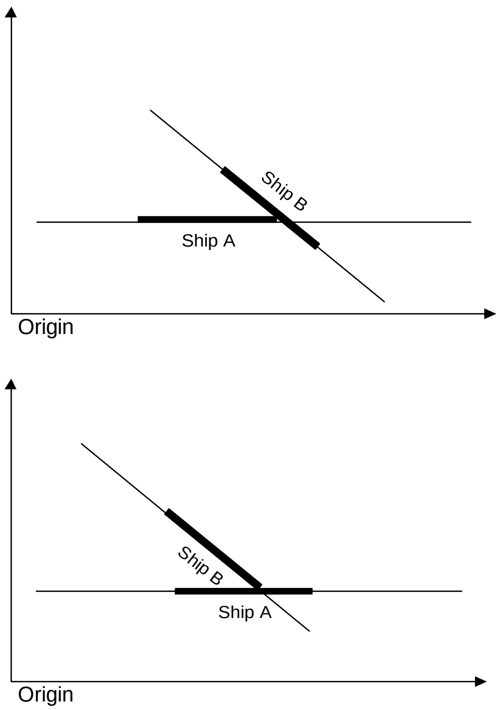


Fig. 3: (a) Ship A hits Ship B and (b) Ship B hits Ship A.

Let the time be T_{hit} when the collision takes place. When Ship A Hits Ship B, the point of hitting at Ship B abaft bow is obtained as,

H = $\sqrt{\{X_C - (x_{Bbow} + V_B \cos\phi \times T_{hit})\}^2 + \{Y_C - (y_{Bbow} + V_B \sin\phi \times T_{hit})\}^2}$ On the other hand when Ship B Hits Ship A, the point of hitting at Ship A abaft bow is given by,

$$H = (x_{Abow} + V_A \times T_{hit}) - X_C$$

Using these mathematical formulations a computer program has been developed in Microsoft (TM) Excel as shown in Fig. 4. A summary of the simulation before collision is also provided in Table 1.

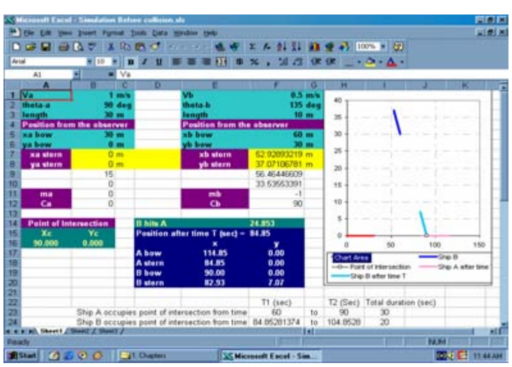


Fig. 4: Developed computer program for analysis before collision.

Table 1: Summary of simulation before collision

With the given speeds and headings, will there be a collision between two ships?	***If the following relations are satisfied*** ***(a)* TA$_{arrival}$ < TB$_{arrival}$ < TA$_{departure}$** ***Or*** (b) $TB_{arrival}$ < $TA_{arrival}$ < $TB_{departure}$
If there is a collision, where will it take place (i.e the location)?	At point C (Xc, Yc)
Which ship strikes the other?	For 1 (a), Ship B hits Ship A For 1 (b), Ship A hits Ship B
At which location the bodies get hit?	H units abaft bow of the struck ship

3.2 Model for Simulation During and After Collision

During a two ship collision the bow of one of the ships hits the other, the first ship may be named the "Striking Ship" and the other ship which is being hit at any place excluding its bow may be called the "Struck Ship". In the case of a head on collision or a collision at the bows (angle of attack is 180 degrees), the vessel with relatively lesser forward speed than the other may be considered as the "Struck Ship" while the ship with relatively greater forward speed may be tagged as "Striking Ship". By considering a collision scenario, as shown in Fig. 5, where Ship A strikes Ship B, two co-ordinate systems may be assumed for each ship such as X-Y for striking ship and I-J for struck ship

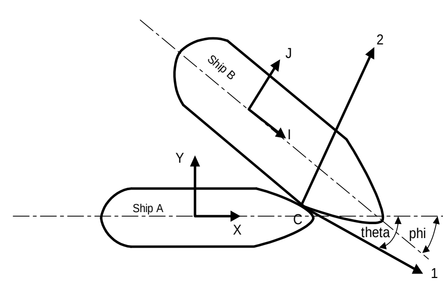


Fig. 5: Co-ordinate system of a ship-ship collision.

The following notations are considered for the development of the model:

c = Point of impact at the collision surface
1 = Direction tangent at the point of impact
2 = Direction normal to 1-axis
X = Direction along centreline of the striking vessel
Y = Direction along the transverse axis of striking vessel
I = Direction along centreline of the struck vessel
J = Direction along the transverse axis of struck vessel
θ = Angle between X axis and 1 axis
φ = Angle between X axis and I axis
M_A = Mass of ship A
M_B = Mass of ship B
V_A = Forward velocity of Ship A
V_B = Forward velocity of Ship B
$V_{A1\ after}$= Velocity of ship A in the direction of 1-axis after collision
$V_{A1\ before}$= Velocity of ship A in the direction of 1-axis before collision
$V_{A2\ after}$ = Velocity of ship A in the direction of 2-axis after collision
$V_{A2\ before}$ = Velocity of ship A in the direction of 2-axis before collision
$V_{B1\ after}$ = Velocity of ship B in the direction of 1-axis after collision
$V_{B1\ before}$ = Velocity of ship B in the direction of 1-axis before collision
$V_{B2\ after}$ = Velocity of ship B in the direction of 2-axis after collision
$V_{B2\ before}$ = Velocity of ship B in the direction of 2-axis before collision
$V_{A1} = V_{B1}$ = Common velocity in the direction of 1-axis after reaching maximum pressure
$V_{A2} = V_{B2}$ = Common velocity in the direction of 2-axis after reaching maximum pressure
E=Co-efficient of restitution, varies in between 0 (perfectly inelastic) to 1 (perfectly elastic)
T_{col}=Collision time, the time required to change the momentum or velocities
Fn = Force in n-axis direction (e.g. FX is force in the direction of X-axis)

3.2.1 The Collision Forces

Using simple trigonometric relations the collision forces in the respective axes on both the struck and striking ship may be computed. For example, forces on Ship A in X-axis and Y-axis direction are,

$$F_X = F_1 \cos\theta + F_2 \cos(90-\theta)$$

$$F_Y = - F_1 \sin\theta + F_2 \sin(90-\theta)$$

Similarly, forces on Ship B in I-axis and J-axis direction are obtained as,

$$F_I = F_1 \cos(\varphi-\theta) + F_2 \cos(\varphi-\theta)$$

$$F_J = F_1 \cos(\varphi-\theta) + F_2 \cos(\varphi-\theta)$$

Here, Forces F_1 and F_2 are perpendicular forces acting at the contact point C developed from the impact between the two bodies. It is known that impact force at a particular direction is equal to change of linear momentum in that direction, i.e.
F_1 = Change in momentum in 1-axis direction
F_2 = Change in momentum in 2-axis direction
Therefore, by using above expressions the forces may be obtained,

For Ship,

$$F_{A1} = M_A \frac{V_{A1after} - V_{A1before}}{T_{col}}$$

$$F_{A2} = M_A \frac{V_{A2after} - V_{A2before}}{T_{col}}$$

For Ship B,

$$F_{B1} = M_B \frac{V_{B1after} - V_{B1before}}{T_{col}}$$

$$F_{B2} = M_B \frac{V_{B2after} - V_{B2before}}{T_{col}}$$

Where,

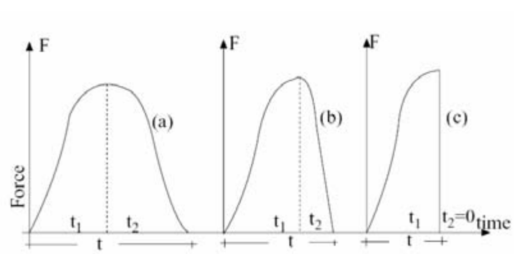
F_{A1} = Force on Ship A in the direction of 1-Axis
F_{A2} = Force on Ship A in the direction of 2-Axis
F_{B1} = Force on Ship B in the direction of 1-Axis
F_{B2} = Force on Ship B in the direction of 2-Axis

3.2.2 The Coefficient of Restitution (E) and Final Velocities

The most fundamental approach of this study is that the changes of velocity of the moving ships after the collision are functions of coefficient of restitution and the time required to restitute or simply the collision time. The application of these two said variables are therefore, very critical and requires careful assumption to model a potentially realistic scenario. The coefficient of restitution is a measure of the elasticity of the collision between two objects. Elasticity is a measure of how much of the kinetic energy of the colliding objects before the collision remains as kinetic energy of the objects after the collision.

There are three types of collision: Perfectly Elastic, Perfectly Inelastic and Elastoplastic collision. A perfectly elastic collision has a coefficient of restitution of 1. Example: two diamonds bouncing off each other. A perfectly inelastic, collision has E = 0. Example: two lumps of clay that don't bounces at all, but stick together. On the other hand an elastoplastic collision, some kinetic energy is transformed into deformation of the material, heat, sound, and other forms of energy. For this type the coefficient of restitution varies be between zero and one.

Oztas [21] discussed elaborately the application of coefficient of restitution with respect to car crashes. Oztas divided the impact Time t into t_1 and t_2, as shown in Figure 3.6. Where t_1 is the compression time and, during this time, the object starts to change its shape and reaches its maximum level, i.e., Force and Reshaping are at a maximum; after this time, the relative velocity between the objects is zero. Again t_2 is the restitution time and simultaneously the reshaping lessens and finally disappears. In the first diagram, $t_1 = t_2 = 1/2t$; it corresponds to an perfectly elastic Collision as referred to in Fig. 6(a). In the second diagram, $t_1 \neq t_2$, and this case is Inelastic, meaning that it is an Elastoplastic Collision, as seen in Fig. 6(b). In the third diagram, t_2 becomes zero ($t_2 = 0$), that means the objects collide and cannot be separated. This impact is a Plastic Collision, Fig. 6(c)



(a) (b) (c)

Fig. 6: Relations between Force and Time during impact [21].

Now when a collision starts taking place the change in momentum is equal to the impulse integral and the common velocity at the beginning of the restitution time reaches the maximum level or quite in other words the velocity reaches maximum at the end of compression. Therefore, according to the impulse momentum theory the following may be obtained for time in between zero to t_1,

For Ship A, Along 1-Axis

$$M_A\left(V_{A1} - V_{A1before}\right) = \int_0^{t_1} F_{B1}dt$$

Along 2-Axis

$$M_A\left(V_{A2} - V_{A2before}\right) = \int_0^{t_1} F_{B2}dt$$

For Ship B, Along 1-Axis

$$M_B\left(V_{B1} - V_{B1before}\right) = \int_0^{t_1} F_{A1}dt$$

Along 2-Axis

$$M_B\left(V_{B2} - V_{B2before}\right) = \int_0^{t_1} F_{A2}dt$$

According to impulse momentum theory,

Along 1-Axis

$$\int_0^{t_1} F_{A1}dt + \int_0^{t_1} F_{B1}dt = 0$$

Along 2-Axis

$$\int_0^{t_1} F_{A2}dt + \int_0^{t_1} F_{B2}dt = 0$$

Thus operating the above relationships the common velocities are obtained,

Along 1-Axis

$$V_{A1} = V_{B1} = \frac{M_B V_{B1\ before} + M_A V_{A1\ before}}{M_B + M_A}$$

Along 2 Axis

$$V_{A2} = V_{B2} = \frac{M_B V_{B2\ before} + M_A V_{A2\ before}}{M_B + M_A}$$

Similarly during the restitution time (between t_1 and t_2) the followings are obtained,

For Ship A, Along 1-Axis

$$M_A\left(V_{A1after} - V_{A1}\right) = \int_{t_1}^{t_2} F_{B1}dt$$

Along 2-Axis

$$M_A\left(V_{A2after} - V_{A2}\right) = \int_{t_1}^{t_2} F_{B2}dt$$

For Ship B, Along 1 axis

$$M_B\left(V_{B1after} - V_{B1}\right) = \int_{t_1}^{t_2} F_{A1}dt$$

Along 2-Axis

$$M_B\left(V_{B2after} - V_{B2}\right) = \int_{t_1}^{t_2} F_{A2}dt$$

According to impulse momentum theory,
Along 1-Axis

$$\int_{t_1}^{t_2} F_{A1}dt + \int_{t_1}^{t_2} F_{B1}dt = 0$$

Along 2-Axis

$$\int_{t_1}^{t_2} F_{A2}dt + \int_{t_1}^{t_2} F_{B2}dt = 0$$

Thus the common velocities are,
Along 1-Axis

$$V_{A1} = V_{B1} = \frac{M_B V_{B1\ after} + M_A V_{A1\ after}}{M_B + M_A}$$

Along 2-Axis

$$V_{A2} = V_{B2} = \frac{M_B V_{B2\ after} + M_A V_{A2\ after}}{M_B + M_A}$$

It is now possible to establish a relationship between impulse integrals with the help of Coefficient of Restitution (*E*). This relation can be expressed as the following:

$$\int_{t_1}^{t_2} Fdt = E\int_{0}^{t_1} Fdt$$

Therefore, it is now possible to obtain the expressions of velocities after collision for both the ships using the above relationship. Such as,

For Ship A:

$$V_{A1after} = V_{A1}\left(1+E\right) - E \times V_{A1before}$$
$$V_{A2after} = V_{A2}\left(1+E\right) - E \times V_{A2before}$$

For Ship B:

$$V_{B1after} = V_{B1}\left(1+E\right) - E \times V_{B1before}$$
$$V_{B2after} = V_{B2}\left(1+E\right) - E \times V_{B2before}$$

The loss of kinetic energy is therefore, obtained according to the following:
For Ship A
Along 1-Axis:

$$KE_{A1} = \frac{1}{2}M_A\left(V_{A1before}{}^2 - V_{A1after}{}^2\right)$$

Along 2-Axis

$$KE_{A2} = \frac{1}{2}M_A\left(V_{A2before}{}^2 - V_{A2after}{}^2\right)$$

For Ship B, Along 1-Axis

$$KE_{B1} = \frac{1}{2}M_B\left(V_{B1before}{}^2 - V_{B1after}{}^2\right)$$

Along 2-Axis:

$$KE_{B2} = \frac{1}{2}M_B\left(V_{B2before}{}^2 - V_{B2after}{}^2\right)$$

3.3 Solution of the Equation of Motion

The equation of motion needs to be solved with necessary boundary conditions in order to find the ships responses due to collision forces. During a collision the equation of motion may be expressed as the following,

$$M_V \frac{d^2x_i}{dt^2} + b_{ij}\frac{dx_i}{dt} + c_{ij}x_i = Fc_i(t)$$

Therefore, the general solution of the equation may be expressed as,

$$x_i = e^{\alpha t}\left\{A_1 \sin\beta t + A_2 \cos\beta t\right\}$$

Where, A_1 and A_2 are constants which are needed to be determined using appropriate boundary conditions.

Assuming an initial condition when the collision force is maximum at time $t = t_{max} = 0$, the displacement is $x_i = x_{i0}$. According to the theory of simple harmonic motion, this amplitude or displacement is maximum when the velocity reaches to zero and the velocity becomes maximum when the amplitude becomes zero units. Therefore, assuming x_{i0} is the maximum amplitude due to collision force at the time $t = 0$, the following unknowns are obtained from the equation of general solution as derived above.

$$A_1 = -\frac{x_{i0}}{\beta} \quad \& \quad A_2 = x_{i0}$$

And therefore, the general solution becomes,

$$x_i = x_{i0}e^{\alpha t}\left\{\cos\beta t - \frac{1}{\beta}\sin\beta t\right\}$$

The above equation is similar to the damping part of any equation of motion where x_{i0} resembles the maximum amplitude due to an excitation and $e^{\alpha t}$ resembles exponential decay of the motion.

3.4 Force as an Exponential Function of Time

The time history of force is considered absolutely vital for solving the equation of motion in time domain. However, experience suggest that in most of the practical cases the force-time data is extremely difficult to predict since it involves complicated internal structural arrangement, including the external fenders, of ship hull that are subject to progressive structural deformations/failures by buckling, shearing, tearing, crushing, bending and twisting of plates, stringers, panels etc during a collision.

Awal [22] proposed several force functions in this aspect but the formulations are yet to be experimentally verified. In this particular study the force is assumed to at an exponential function of time where the force increases exponentially from time t_{hit} to time t_{max} and thereafter it reduces exponentially again from time t_{max} to time t_{sep} as shown in Fig.7.

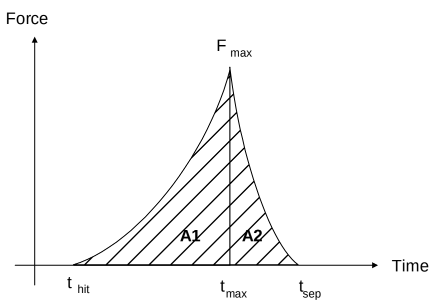


Fig. 7: Collision force is an exponential function over the contact period.

The particular integral of the function $Fc_i(t) = F_{\max i} f(e^t)$ may be obtained as the following,

$$x_i = \frac{F_{\max}\left[e^{t-t\max}\right]}{a_{ij} + b_{ij} + c_{ij}} \qquad [t = t_{hit} \text{ to } t = t_{max}]$$

3.5 Validation of the Model

The developed model has been compared with a number of published research works which are described in the following paragraphs.

3.5.1 Comparison of Lost Kinetic Energy

The comparison of loss of kinetic energy has been done using two similar ships of length 116 meter (particulars are given in Table 2). The collisions were taken at various angles of attack and speeds as well. The results are compared with published data of Petersen [41], Hanhirova [42] and Zhang [15] as shown in Table 3. It is however, considered in this validation that the hitting takes in place at the centre of the struck ship and the collision is entirely plastic. A plastic collision means that the ships remain in contact after the collision and all the kinetic energy is being used in deforming the ships hull structure and dynamic movement of the ships.

Table 2 Vessel and collision particulars

Length	116 meters
Breadth	19.0 meters
Draft	6.9 meters
Displacement	10,340 ton
Coefficient of restitution	0.0

The results are being compared with the loss of kinetic energy along 1-axis (KE1) and 2-axis (KE2) directions (defined earlier) and the units expressed here are in mega joule. The comparison suggests that there are noticeable variations among different methods adopted by different researcher; nevertheless, the results do not exceed the comparative limits and thus it may be concluded that the developed model is in good agreement.

Table 3: Comparison of loss of kinetic energy

Va (m/s)	Vb (m/s)	α=β (deg.)	KE1 (MJ)				KE2 (MJ)			
			rsen (198	va	ng (199	ent Stud	rsen (198	va	ng (199	ent Stud
4.5	0	90	0	0	0	0	69.6	54.4	70.1	78.519
4.5	4.5	90	24.7	41.5	21.4	26.173	64.1	54.4	70.1	78.519
4.5	4.5	60	5.2	15.8	0.2	32.716	28.8	28.3	35.3	58.889
4.5	4.5	30	49.3	7.2	0	12.616	71.9	4	7.4	19.629
4.5	0	120	9.8	14	15	19.629	54	40.9	50.1	58.889
4.5	2.25	120	40.7	51.5	45.1	26.173	60.3	42.8	57.5	58.889

3.5.2 The Hydrodynamic Coefficients

The hydrodynamic coefficients a_{ij}, b_{ij} and c_{ij} depend on the hull form and the interaction between the hull and surrounding water. The coefficients may also vary during a collision as well and the range of variation is even wider considering open or restricted water conditions. However, for simplicity Minorsky (1959) proposed to use a constant value of the added mass coefficients of ships for the sway motion, $m_{ay} = a_{22} = 0.4$. Motora et.al [23] conducted a series of tests in determining the added mass coefficients for sway motion and has found that the coefficient varies in the range of 0.2 to 1.3. The study revealed that the longer the duration, the larger the value of the coefficient. The added mass coefficient related to forward motion is found to be relatively smaller and in the range of, $m_{ax} = a_{11} =$ 0.02 to 0.07. The added mass coefficient for rolling is suggested by Bhattacharyya [24] to be in between 10 to 20 percent of the actual displacement of the ship. The added mass coefficient for yaw motion of the ship, j_a, is used by Pedersen et. al (1993) as 0.21. Crake [25] in his research work used an empirical relation for finding the added mass in yaw as $a_{66} = 0.0991\rho T^2 L_{BP}^3$. However, in this study the hydrodynamic coefficients were determined using the 3-D source distribution method [26] and the values are compared with existing results expressed in range of virtual mass in Table 4.

It is observed from the comparison that the hydrodynamic coefficients for surge, sway and yaw fairly matches within the range except a few discrepancies in the sway motion. This is probably because the range is determined on the basis of ships that are relatively large and ocean going in comparison to the small vessels designed for inland transportation in Bangladesh.

Table 4: Comparison of virtual mass

Hydrodynamic Coefficients	Range of Virtual Mass (non dimensional)	46 m Vessel (3-D Method)	32 m Vessel (3-D Method)
Surge, a_{11}	1.02 – 1.07	1.01	1.01
Sway, a_{22}	1.20 – 2.30	1.05	1.14
Roll, a_{44}	1.10 – 1.20	1.61	1.22
Yaw, a_{66}	1.20 – 1.75	1.40	1.53

4 Results and Discussions

This section reveals the results obtained from the numerical investigations and discusses the various facts revealed from the analyses. In the present study two different vessels (principle particulars are given in Table 5) were considered for conducting numerical investigation in different scenarios. Both the vessels are common inland vessels and such hull shapes are generally been used both in cargo and passenger ships. Results are presented in two different categories, (1) Analysis of non dimensional forces due to changes in different variables and (2) Time domain simulation due to changes in different variables.

Table 5: Principle particulars of the ships

Principle Particulars		
Ship 1 (46 Meter)	Length	46.800 meter
	Breadth	10.564 meter
	Draft	2.3340 Meter
	Displacement	556.3 Tone
	Angle of vanishing stability	67 degree
Ship 2 (32 Meter)	Length	30.640 meter
	Breadth	6.700 meter
	Draft	3.500 Meter
	Displacement	498.0 Tone
	Angle of vanishing stability	75 degree

The numerical model setup consists of two ships positioned perpendicularly and potentially striking at amidships as shown in Fig.8. The variables considered in the study are the speed of the striking ship, collision time and added mass for sway direction. An important aspect to observe is the point of blow above the water level through which the collision force acts largely depends on the bow profile and the midship section of the ships. However, for this particular study it is assumed that the collision contact point is ½ draft above the centre of gravity.

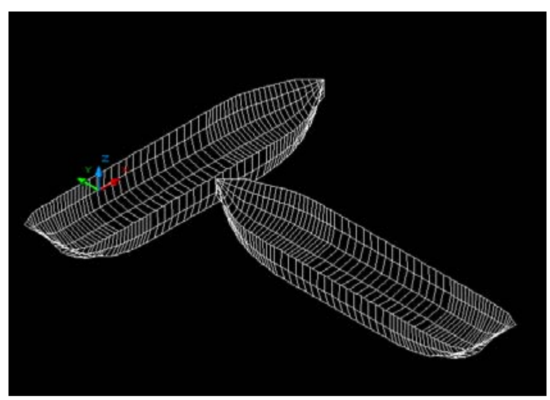


Fig. 8: Model setup showing striking at 90 degree angle of attack at amidships.

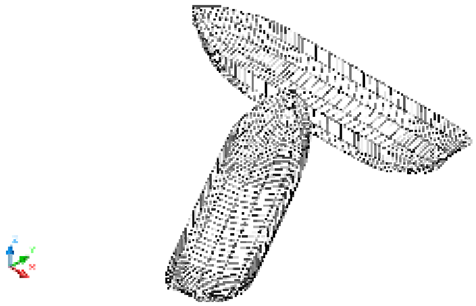


Fig. 9: A typical 3D Mesh collision of two ships

This study considers typical inland vessel of length 10.64-meter and 46.8 meters for struck vessel and striking vessel respectively. Fig. 9 depicts a typical collision scenario generated using 3D mesh for this particular study. The breadth and depth of the struck vessel are 6.7 meter and 3.5 meter respectively. At full load the displacement of the struck ship is 498 tones and the angle of vanishing stability is around 63 degrees in still water condition as shown in Fig. 10.

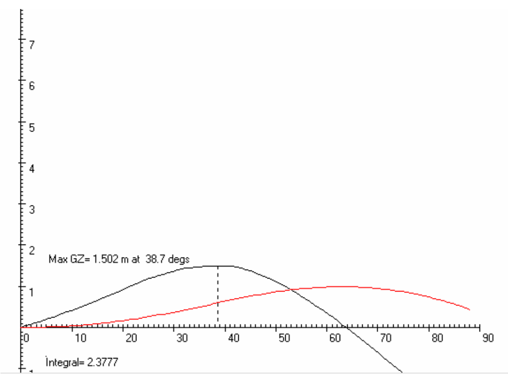


Fig.

Fig. 10: Hydrostatic roll stability of the struck ship

4.1 Analysis on Forces

It is revealed in Fig. 11 that the force in sway direction varied in a range up to 6 times of the displacement or buoyancy force due to variation in striking ship speed. The speed is taken up to 12 knots (6.1782 m/sec) since generally this is the maximum allowable speed in the inland waterways.

Graphically, the higher the speed of the striking ship, the higher the force in sway direction. Thus, the force in sway direction is proportional with the striking ship speed. It is also observed that at higher speeds the coefficient of restitution with lower value plays a very important role by reducing the collision force significantly.

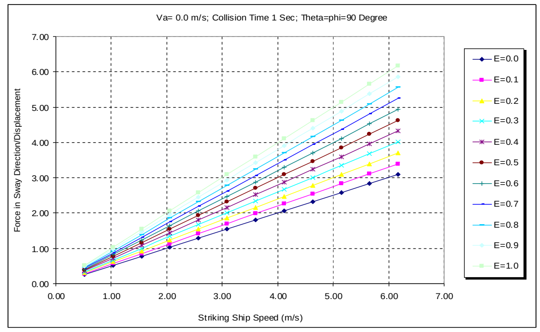


Fig. 11: Force in sway direction vs. striking speed at various restitutions.

Fig. 12 suggests that the collision force substantially decreases with the increase in collision/contact period. For a very short collision period the variations of force in the sway direction due to different coefficients of restitution are very large but for a shorter collision period the variations are comparatively small and practically trivial.

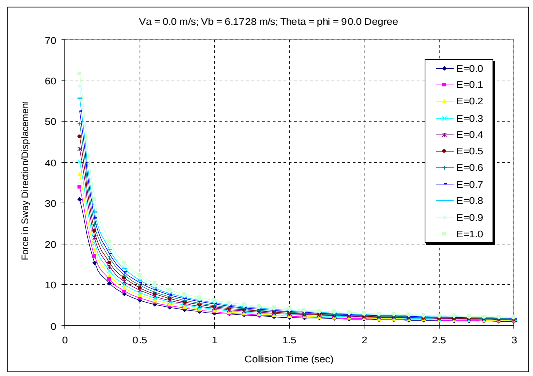


Fig. 12: Force in sway direction vs. collision time at various restitutions.

The collision force is also notably affected by the added mass of the ship. It is observed from Fig. 13 that higher the added mass the lower is the collision force and vice versa. It is observed from the figure that for lower added mass the force varies relatively largely with the variation in coefficient of restitution while the force varies relatively less at higher added mass.

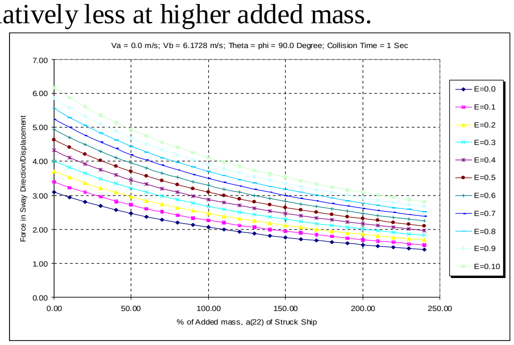


Fig. 13: Force in sway direction vs. added mass for various restitutions.

The variation in collision angle also significantly affects the collision force. Fig. 14 and Fig. 15 show the non-dimensional force in surge and sway direction respectively for various coefficients of restitutions and angle of attack. In this particular graph the striking ship speed is assumed to be 12 knots and the speed of the struck ship is assumed to be 0 (zero) knot. The collision duration is taken to be 1 second.

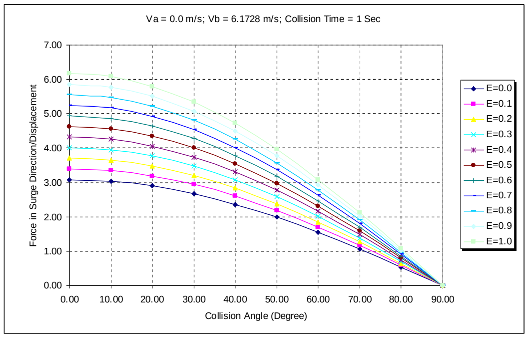


Fig. 14: Force in surge direction for various collision angle & restitution.

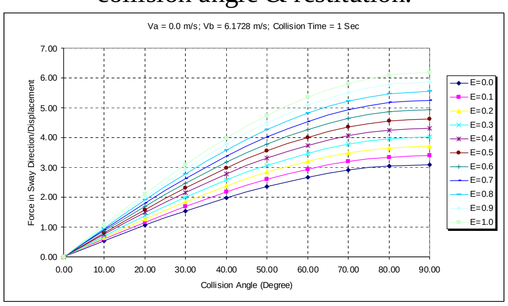


Fig. 15: Force in sway direction for various collision angle & restitution.

4.2 Results on Maximum Amplitude

Attempt has been taken to investigate the capsizing phenomena under two variables at a time as shown in the following figures. Fig. 16 represents the maximum rolling angle against two different variables namely, the collision time and collision angle. Collision time refers to the time required for contact, compression, restitution and separation of the contact surfaces of the ships. It has been observed from the chart that the relation between collision time and rolling amplitude is exponential and the relation between collision angle and roll amplitude is trigonometric.

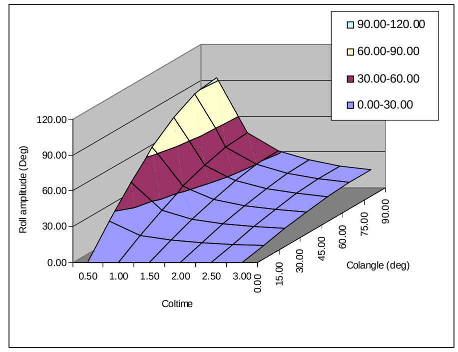


Fig. 16: Maximum roll amplitude against collision time and collision angle.

The surfaces, as shown by separate colours, represent a particular region for range of rolling amplitude where collision time and collision angle are the variables. In order to find a safe condition, the first and most important parameter is that the rolling amplitude doesn't exceed the angle of vanishing stability. For example in this figure the top two surfaces (roll angle 60 to 90 and roll angle 90 to 120) represent the unsafe condition which exists roughly in the range of 40 deg to 90 deg of collision angle and zero to around 0.8 second of collision time. Therefore, any surface excluding these boundaries represents a safer situation while other variables are kept constant. In this case of Fig. 17, the ship may survive a collision if it operates or designed with the following parameters: (a) Collision angle is in between zero to 40 degrees, (b) collision time roughly greater than 0.8 second.

Fig. 18 represents a 3D surface chart for a range of coefficient of restitutions (E) and speeds of the striking ship (Vb). It is observed that both the variables influence the rolling amplitude linear proportionally. The range for safe surface is observed to exist in between the following variables: (a) 0 to 0.62 of coefficient of restitution and (b) striking speed less than 2.5 meters/second (5 knots).

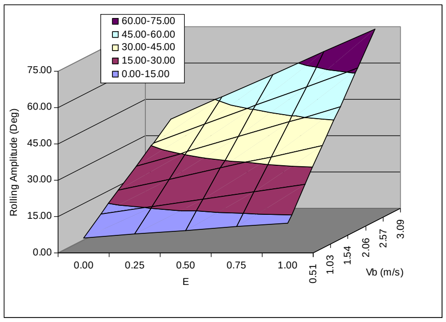


Fig. 17: Maximum roll amplitude against coefficient of restitution and striking ship's speed.

Fig. 18 shows a surface of collision angle and height of contact point above the centre of flotation. It is observed that the higher the position of the contact point the higher is the amplitude of role while others variables are kept constant, The amplitude, however, reduces quite significantly with the decrease in the vertical position of the contact point. However, the range of safe surface exists between the following ranges: (a) Collision angle less than 35 degrees and (b) height of collision contact point below 1.75 meters measured above centre of floatation.

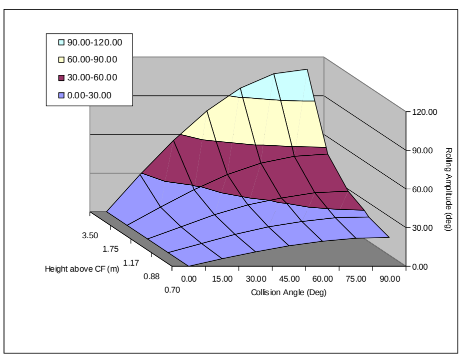


Fig.18: Maximum roll amplitude against collision angle and height of contact point above centre of floatation.

4.3 Time Domain Simulation for Different Cases

The time domain simulation of collision between ships depicts clear picture of the dynamic responses of ships and provides explicit means for comprehending the total scenario. This particular study investigates seven different cases which have been summarised in the Table 6. Fig. 19, Fig. 20 and Fig. 21 represents the time domain roll simulation of the struck ships. It is observed that higher striking speeds cause higher the moment for rolling and thus higher rolling amplitude. Although this phenomena is nonetheless a common fact but the key aspect is to observe the amplitudes which are being reduced significantly by alteration of the coefficient of restitution.

Table 6: Summary of the collision scenarios investigated in this study.Case

No.	Name of the Struck Ship	Name of the Striking Ship	Speed of the Striking Ship (knots)
1	46 Meter Vessel	46 Meter Vessel	1.0
2	46 Meter Vessel	46 Meter Vessel	6.0
3	32 Meter Vessel	32 Meter Vessel	1.0
4	32 Meter Vessel	32 Meter Vessel	6.0
5	32 Meter Vessel	46 Meter Vessel	1.0
6	32 Meter Vessel	46 Meter Vessel	3.0
7	32 Meter Vessel	46 Meter Vessel	6.0

It is observed that up to 83 percent of the rolling amplitude may be reduced if zero restitution materials are being used. This is indeed, a very important aspect of the research findings that as excessive rolling causes ships to capsize and such capsizing could be prevented by applying the lower restitution shock absorbing materials. This phenomena is simulated in Fig. 20 (6 knot) and Fig. 21 (3 and 6 knot) where the ships roll over the angle of vanishing stability if the coefficient of restitutions are one for all the cases. These roll amplitudes, however, are significantly less in their respective cases if the coefficient of restitutions were considered zero or close to zero. Therefore, the facts revealed here could be a mater of life and death and indeed requires due importance to be looked into while construction ship fenders and other similar protective devices.

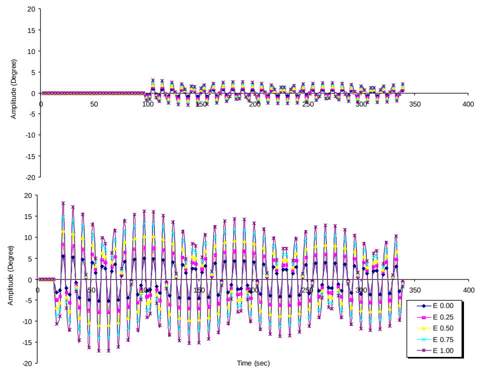


Fig. 19: Time domain simulation for rolling motion for 48m vessel vs 48 m vessel at 90 degrees amidship for speeds 1 knot and 6 knot.

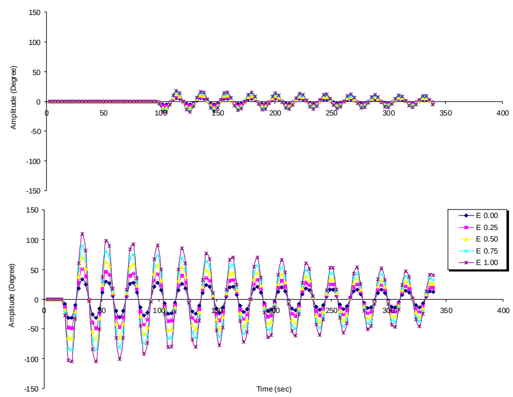


Fig. 20: Time domain simulation for rolling motion for 32m vessel vs 32 m vessel at 90 degrees amidship for speeds 1 knot and 6 knot.

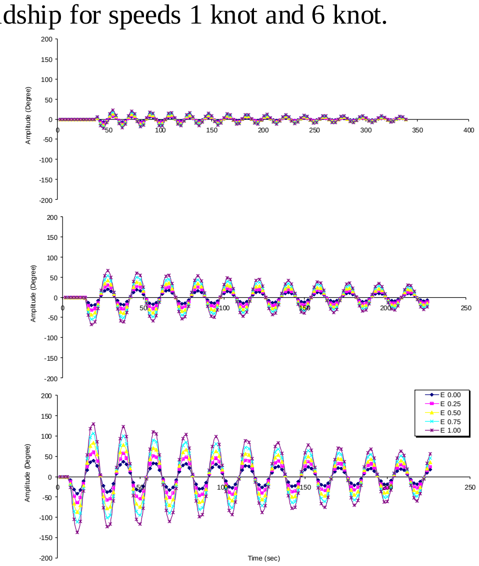


Fig. 21: Time domain simulation for rolling motion for 32m vessel vs 48 m vessel at 90 degrees amidship for speeds 1 knot, 3 knot and 6 knot.

5 Conclusions and Recommendations

5.1 Concluding Remarks

The research on studying the dynamic behaviour of ships for different coefficient of restitutions of the hull material is indeed a new concept and so far limited knowledge is available to the researchers in this particular area. Therefore, as a preliminary investigation the study is limited to mathematical formulations only and due to deficiency in infrastructural facilities for experimental investigations the study had no other options but to validate its results with published numerical results. Based on the results, it could be concluded that vessels of this particular type plying in relatively calm waters may use fenders made of materials having coefficient of restitution less than 0.5 and restitution time greater than 1 second to avoid consequences due to collision with a similar vessel. In addition, the collision angle has to be less than 33 degrees, the relative struck speed has to be less than 2.5 meters per second and the vertical collision contact point has to be less than 1.75 meters above the centre of floatation.

The research on studying the capsizing of ships due to collision is still in the initial developing stage. The application of the parameter of coefficient of restitution of the hull material is considered fundamentally new. So far limited knowledge is available to the researchers about its affect on ship’s dynamic behaviour. Further research on this model is therefore recommended, as it seems highly potential in terms of suggesting survivability boundaries for various operating conditions.

5.2 Recommendations

Based on the knowledge obtained from this particular study the following recommendations can be drawn:

- The materials used in the construction of fenders and other external protective devices or structural members are recommended to have as much lower coefficient of restitution as possible. Such adaptation significantly reduces the collision forces imparted on the dynamics of ships in events of high speed collision.
- Due to practical limitations, if materials with lower restitutions do not get the economical feasibility, the compensation may be made by using materials that have longer duration for restitution. This will reduce the collision force significantly although it may restitute hundred

percent after being compressed due to external striking force but the change in momentum will be decreased due to larger value of time which will result reduced collision force on the ship. This type of fender may be economically feasible for using in ships over a longer period.

- Under the prevailing situation, it might be a cost effective measure to impose restrictions on some vessels in plying highly congested water areas, particularly boats those are unable to withstand collision. It is observed that the smaller boats in Bangladesh are mostly used for shorter trips and carry excessive passengers (e.g. Kheya Boats) and when these vessels encounter an accident they cause a high number of fatalities. Therefore, new legislations may be activated on collision worthiness particularly giving emphasis on smaller boats and cargo ships as well.
- It is observed that there is a much more scope of research in this particular are of collision dynamics. Future study may be conducted on smaller country boats and with more detailed description of the waterways as the river width, depth, geometry etc which may play a notable role in such investigations. Further study with wave and wind consideration is also recommended as it may influence the ships dynamics either adversely or favourably during a collision. Experimental investigations are indeed recommended for future study.

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