

A STUDY ON SHIP CAPSIZING DUE TO COLLISION

Zobair Ibn Awal¹, M. Rafiqul Islam^{2*}

¹Department of Naval Architecture & Marine Engineering
Bangladesh University of Engineering & Technology (BUET)
Dhaka 1000, BANGLADESH

²Faculty of Mechanical Engineering
Universiti Teknologi Malaysia
81310 UTM, Skudai, Johor, MALAYSIA

ABSTRACT

This paper investigates the ship capsizing due to collision with another ship in calm water. A mathematical model on collision dynamics has been developed and validated. The dynamic characteristic (roll in particular) has been studied numerically. Searches have been made to find the survivability boundaries in terms of striking velocity, coefficient of restitution, collision angle, collision time and vertical position of hitting point. Maximum amplitude of rolling motion has been determined against the said parameters and have been presented as 3D surface charts which enable to identify safe boundaries of operation for two given variables at a time. This particular approach of collision simulation has the potential of analysing operational risks during a ship's design stage and thereby incorporates necessary modifications if required. Recommendations have been put forward for future studies.

Keywords: *Ship capsize, simulation, collision dynamics, rolling*

1.0 INTRODUCTION

Many maritime countries around the world frequently encounter problems of ship collision quite often both in the inland waterways and in the seas as well. Ship collisions are of particular importance due to the following reasons:

- i) The environmental impact, especially in the case where large tanker ships are involved. However, even minor spills from any kind of merchant ship can create a threat to the environment.
- ii) The loss of human life is invaluable.
- iii) Financial consequences to local communities close to the accident site and consequences to ship-owners, due to ship loss or penalties.

The increase in business activity in many developing countries has resulted in denser sea routes and at the same time faster ships for quick transportation.

* Corresponding author: E-mail: rafiquilis@fkm.utm.my

Therefore, the possibilities that a ship may experience a major accident during her lifetime are higher. Denser sea routes increase the probability of an accident, collision in particular, involving ships or offshore structures. Due to extremely large masses and relatively high velocities the energy involved in such an accident is astonishing. An event like this may confront a ship to sustain severe structural damage and capsizing as well.

Over the years there have been numerous cases of ship capsizing and collision accidents. A study by Awal, Islam and Hoque [1] on inland shipping accidents reveals that around 40 percent of total accidents in Bangladesh occur due to collision. Similarly, studies on ship capsizing exhibit that the disasters are recurring quite frequently around the world as well. A number of North Sea trawlers from the UK and other EU countries have capsized in heavy seas in the past thirty years. At least one RO-RO vessel becomes a casualty in each week [2, 3]. In Canada the average number of accidents due to collision and capsizing is around 45 per year [4].

2.0 BACKGROUND AND SCOP OF STUDY

The problem of capsizing has been generally treated as a phenomenon originating from wave and wind forces. A number of in depth research has been conducted over the years to understand the capsizing mechanisms and ways to prevent it [5, 6].

Lin & Yim [7] used the new subject of chaos to analyse the non-linear equations devised to represent the motion of ships in roll-sway coupled motions. They discussed four types of capsize:

- i) Non-oscillatory capsizing in which the restoring moment is small compared with the moments of wind and waves exerted on the ship.
- ii) Oscillatory sudden capsizing in this case restoring moment should be sufficient but instability is caused by successive series of waves.
- iii) Oscillatory symmetric build-up capsizing, here amplitudes of rolling motion increase rapidly after only a few cycles similar to linear resonance. The build-up is likely to be caused by a series of waves.
- iv) Oscillatory anti-symmetric build-up capsizing. In some cases the rolling motion appears to be antisymmetric with respect to the axis of symmetry about the time axis. This again appears as the result of passing through a succession of waves producing oscillations, which are so large that recovery is impossible.

One of the most serious incidents was illustrated by Car Ferry Wahine disaster in 1968, as described by Conolly [8]. The type of disaster is referred to as broaching. In this case the ship is travelling with a stern sea slightly to one quarter. The ship experience difficulty with the rudders being increasingly ineffective. Large yaw angles will be experienced and the ship will roll through a large angle to leeward. The ship is said to be 'broached-to' and the breaking waves over the ship and the wind effects may be sufficient to capsize. Spyro [9] has analysed a phenomenon known as surf-riding where the ships is stationary relative to the wave trough. The simulation is an entrapment of the vessel for prolonged periods

at exactly zero frequency. The author showed that how with the fixed control system the ship becomes unstable and results capsizing.

However, study on capsizing due to collision has yet to be fully understood and published literatures on this topic are rare. Recently, Awal and et. al. [10, 11, 12] have conducted a research where such an attempt has been taken, yet the further developments are needed. Therefore, in this study it has been attempted to investigate capsizing behaviour of a ship which is encountering a collision with another similar vessel. The study searches the survivability boundaries in terms of striking velocity, coefficient of restitution, collision angle, collision time and vertical position of hitting point.

3.0 THEORITICAL MODEL FOR SHIP COLLISION DYNAMICS

Considering a collision scenario, as shown in Figure 1, where Ship B strikes Ship A, two co-ordinate systems may be assumed for each ship such as $X-Y$ for striking ship and $I-J$ for struck ship.

Using simple trigonometric relations the collision forces in the respective axes on both the struck and striking ship may be computed. For example, forces on Ship A in X -axis and Y -axis direction are,

$$F_X = F_1 \cos \theta + F_2 \cos (90-\theta) \quad (1)$$

$$F_Y = -F_1 \sin \theta + F_2 \sin (90-\theta) \quad (2)$$

Similarly, forces on Ship B in I -axis and J -axis direction are obtained as,

$$F_I = F_1 \cos (\varphi-\theta) + F_2 \cos (\varphi-\theta) \quad (3)$$

$$F_J = F_1 \cos (\varphi-\theta) + F_2 \cos (\varphi-\theta) \quad (4)$$

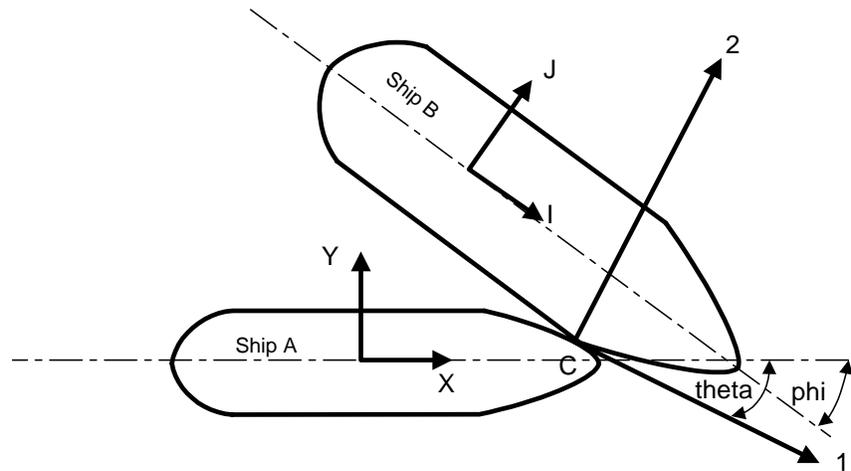


Figure 1: Co-ordinate system of a ship-ship collision

Here, forces F_1 and F_2 are perpendicular forces acting at the contact point C developed from the impact between the two bodies. It is known that impact force at a particular direction is equal to change of linear momentum in that direction, i.e. F_1 equals the change in momentum in 1-axis direction and F_2 equals change in momentum in 2-axis direction. Therefore, by using these expressions the forces may be obtained,

For Ship A,

$$F_{A1} = M_A \frac{V_{A1after} - V_{A1before}}{T_{col}} \quad (5)$$

$$F_{A2} = M_A \frac{V_{A2after} - V_{A2before}}{T_{col}} \quad (6)$$

For Ship B,

$$F_{B1} = M_B \frac{V_{B1after} - V_{B1before}}{T_{col}} \quad (7)$$

$$F_{B2} = M_B \frac{V_{B2after} - V_{B2before}}{T_{col}} \quad (8)$$

3.1 Application of Coefficient of Restitution

The most fundamental approach towards solving this problem is considering the ship velocity after collision as a function of coefficient of restitution and the time required to reconstitute or simply the collision time. The application of these two variables are however, very critical and requires careful assumption to model a potentially realistic scenario. The coefficient of restitution is a measure of the elasticity of the collision between two objects. Elasticity is a measure of how much of the kinetic energy of the colliding objects before the collision remains as kinetic energy of the objects after the collision.

There are three types of collision: Perfectly Elastic, Perfectly Inelastic and Elastoplastic collision. A perfectly elastic collision has a coefficient of restitution of 1. Example: two diamonds bouncing off each other. A perfectly inelastic collision has $E = 0$. Example: two lumps of clay that don't bounce at all, but stick together. On the other hand an elastoplastic collision, some kinetic energy is transformed into deformation of the material, heat, sound, and other forms of energy. For this type the coefficient of restitution varies between zero and one. Now when a collision starts taking place the change in momentum is equal to the impulse integral and the common velocity at the beginning of the restitution time reaches the maximum level or in other words the velocity reaches maximum at the end of compression. Therefore, according to the impulse momentum theory the following may be obtained for time between start of collision ($t=0$) and maximum compression ($t=t_1$),

For Ship A,

$$M_A (V_{A1} - V_{A1before}) = \int_0^{t_1} F_{B1} dt \quad (9)$$

$$M_A (V_{A2} - V_{A2before}) = \int_0^{t_1} F_{B2} dt \quad (10)$$

For Ship B,

$$M_B (V_{B1} - V_{B1before}) = \int_0^{t_1} F_{A1} dt \quad (11)$$

$$M_B (V_{B2} - V_{B2before}) = \int_0^{t_1} F_{A2} dt \quad (12)$$

However, according to impulse momentum theory the following relations must satisfy along the axes,

$$\int_0^{t_1} F_{A1} dt + \int_0^{t_1} F_{B1} dt = 0 \quad (13)$$

$$\int_0^{t_1} F_{A2} dt + \int_0^{t_1} F_{B2} dt = 0 \quad (14)$$

Thus operating the above relationships the common velocities are obtained along the two axes,

$$V_{A1} = V_{B1} = \frac{M_B V_{B1 before} + M_A V_{A1 before}}{M_B + M_A} \quad (15)$$

$$V_{A2} = V_{B2} = \frac{M_B V_{B2 before} + M_A V_{A2 before}}{M_B + M_A} \quad (16)$$

Similarly between the maximum compression and full separation of the ships the followings relations are obtained for common velocities,

$$V_{A1} = V_{B1} = \frac{M_B V_{B1 after} + M_A V_{A1 after}}{M_B + M_A} \quad (17)$$

$$V_{A2} = V_{B2} = \frac{M_B V_{B2 after} + M_A V_{A2 after}}{M_B + M_A} \quad (18)$$

It is now possible to establish a relationship between impulse integrals with the help of Coefficient of Restitution (E). This relation can be expressed as the following:

$$\int_{t_1}^{t_2} F dt = E \int_0^{t_1} F dt \quad (19)$$

Using equation (9) to (19) it is now possible to obtain the expressions of velocities after collision for both the ships. Such as,

$$V_{A1after} = V_{A1}(1 + E) - E \times V_{A1before} \quad (20)$$

$$V_{A2after} = V_{A2}(1 + E) - E \times V_{A2before} \quad (21)$$

$$V_{B1after} = V_{B1}(1 + E) - E \times V_{B1before} \quad (22)$$

$$V_{B2after} = V_{B2}(1 + E) - E \times V_{B2before} \quad (23)$$

3.2 Loss of Kinetic Energy

The loss of kinetic energy is therefore obtained as,

Ship A,

$$KE_{A1} = \frac{1}{2} M_A (V_{A1before}^2 - V_{A1after}^2) \quad (24)$$

$$KE_{A2} = \frac{1}{2} M_A (V_{A2before}^2 - V_{A2after}^2) \quad (25)$$

Ship B,

$$KE_{B1} = \frac{1}{2} M_B (V_{B1before}^2 - V_{B1after}^2) \quad (26)$$

$$KE_{B2} = \frac{1}{2} M_B (V_{B2before}^2 - V_{B2after}^2) \quad (27)$$

3.3 Solution of the Equation of Motion

The equation of motion is required to be solved with necessary boundary conditions in order to find the ships responses due to collision forces. During a collision the equation of motion may be expressed as the following,

$$M_V \frac{d^2 x_i}{dt^2} + b_{ij} \frac{dx_i}{dt} + c_{ij} x_i = F c_i(t) \quad (28)$$

Therefore, the general solution of the equation may be expressed as,

$$x_i = e^{\alpha t} \{ A_1 \sin \beta t + A_2 \cos \beta t \} \quad (29)$$

Where, A_1 and A_2 are constants which are needed to be determined using appropriate boundary conditions. Assuming an initial condition when the collision force is maximum at time $t = t_{max} = 0$, the displacement is $x_i = x_{i0}$. According to the theory of simple harmonic motion, this amplitude or displacement is maximum

when the velocity reaches to zero and the velocity becomes maximum when the amplitude becomes zero units. Therefore, assuming x_{i0} is the maximum amplitude due to collision force at the time $t = 0$, the following unknowns are obtained from the equation of general solution as derived above.

$$A_1 = -\frac{x_{i0}}{\beta} \text{ and } A_2 = x_{i0} \quad (30,31)$$

And therefore, the general solution becomes,

$$x_i = x_{i0} e^{\alpha t} \left\{ \cos \beta t - \frac{1}{\beta} \sin \beta t \right\} \quad (32)$$

The above equation is similar to the damping part of any equation of motion where x_{i0} resembles the maximum amplitude due to an excitation and $e^{\alpha t}$ resembles exponential decay of the motion. It is, however, important to mention that in this paper only rolling motion (x_4) is being investigated to study the capsizing phenomena of the vessels under collisions.

3.4 Forces as an Exponential Function of Time

The time history of force is considered vital for solving the equation of motion in time domain. However, experience suggest that in most of the practical cases the force-time data is extremely difficult to predict since it involves complicated internal structural arrangement, including the fenders, of hull that are subject to progressive structural deformations/failures by buckling, shearing, tearing, crushing, bending and twisting of plates, stringers, panels etc. Awal [10] proposed several force functions in this aspect but the formulations are yet to be experimentally verified. In this particular study the force is assumed to be an exponential function of time where the force increases exponentially from time t_{hit} to time t_{max} and thereafter it reduces exponentially again from time t_{max} to time t_{sep} as shown in Figure 2.

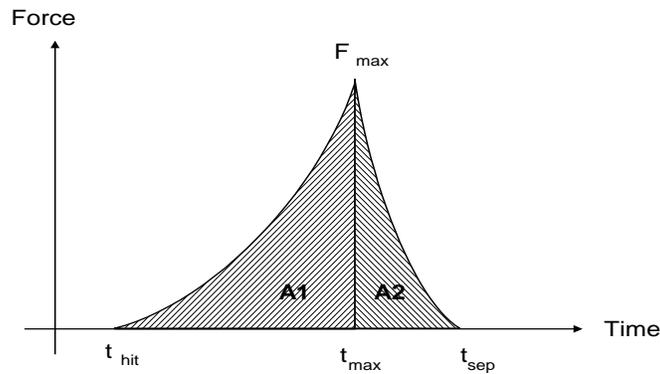


Figure 2: Collision force is an exponential function over the contact period

The particular integral of the function $F_{c_i}(t) = F_{\max i} f(e^t)$ may be obtained as the following,

$$x_i = \frac{F_{\max} \left[e^{t-t_{\max}} \right]}{a_{ij} + b_{ij} + c_{ij}} \quad [t = t_{hit} \text{ to } t = t_{\max}] \quad (33)$$

3.5 Validation of the Model

The developed model has been compared with a number of published research works which are described in the following paragraphs.

3.5.1 Comparison of Lost Kinetic Energy

The comparison of loss of kinetic energy has been computed using two similar ships of length 116 meter. The particulars are breadth 19.0 meters, draft 6.9 meters, displacement 10,340 tons and coefficient of restitution zero. The collisions were at various angles of attack and speeds as well. For validation it is considered that the hitting takes in place at the midship of the struck ship and the collision is entirely plastic. A plastic collision means that the ships remain in contact after the collision and all the kinetic energy is being used in deforming the ships hull structure and dynamic movement of the ships. The results are compared with the loss of kinetic energy along 1-axis (KE1) and 2-axis (KE2) directions and the units expressed here are in mega joule. The results are compared with published data of Petersen [13], Hanhirova [14] and Zhang [15] as shown in Table 1. The comparison suggests that there are noticeable variations among different methods adopted by different researcher; nevertheless, the results do not exceed the comparative limits and thus it may be concluded that the developed model is in good agreement.

Table 1: Comparison of loss of kinetic energy

| Va (m/s) | Vb (m/s) | $\alpha=\beta$ (deg.) | KE1 (MJ) | | | | KE2 (MJ) | | | |
|-------------|-------------|--------------------------|--------------------|---------------------|-----------------|------------------|--------------------|---------------------|-----------------|------------------|
| | | | Petersen (1982) | Hanhirova (1995) | Zhang (1999) | Present Study | Petersen (1982) | Hanhirova (1995) | Zhang (1999) | Present Study |
| 4.5 | 0 | 90 | 0 | 0 | 0 | 0 | 69.6 | 54.4 | 70.1 | 78.52 |
| 4.5 | 4.5 | 90 | 24.7 | 41.5 | 21.4 | 26.17 | 64.1 | 54.4 | 70.1 | 78.52 |
| 4.5 | 4.5 | 60 | 5.2 | 15.8 | 0.2 | 32.72 | 28.8 | 28.3 | 35.3 | 58.89 |
| 4.5 | 4.5 | 30 | 49.3 | 7.2 | 0 | 12.62 | 71.9 | 4 | 7.4 | 19.63 |
| 4.5 | 0 | 120 | 9.8 | 14 | 15 | 19.63 | 54 | 40.9 | 50.1 | 58.89 |
| 4.5 | 2.25 | 120 | 40.7 | 51.5 | 45.1 | 26.17 | 60.3 | 42.8 | 57.5 | 58.89 |

3.5.2 The Hydrodynamic Coefficients

The hydrodynamic coefficients a_{ij} , b_{ij} and c_{ij} depend on the hull form and the interaction between the hull and surrounding water. The coefficients may also vary during a collision as well and the range of variation is even wider considering open or restricted water conditions. However, for simplicity Minorsky [16] proposed to use a constant value of the added mass coefficients of ships for the sway motion, $m_{ay} = a_{22} = 0.4$. The added mass coefficient for rolling is suggested by Bhattacharyya [17] to be in between 10 to 20 percent of the actual displacement of the ship. However, in this study the hydrodynamic coefficients

were determined using the 3-D source distribution method [18] and the values are compared with existing results expressed in range of virtual mass (Table 2). It is observed from the comparison that the hydrodynamic coefficients for surge sway and yaw fairly matches within the range except a few discrepancies in the sway motion. This is probably because the range is determined on the basis of ships that are relatively large and ocean going in comparison to the small vessels designed for inland transportation.

Table 2: Comparison of virtual mass (non dimensional)

| Hydrodynamic Coefficients | Range of Virtual Mass | 46 m Vessel (3-D Method) | 30 m Vessel (3-D Method) |
|---------------------------|-----------------------|-----------------------------|-----------------------------|
| Surge, a_{11} | 1.02 – 1.07 | 1.01 | 1.01 |
| Sway, a_{22} | 1.20 – 2.30 | 1.05 | 1.14 |
| Roll, a_{44} | 1.10 – 1.20 | 1.61 | 1.22 |
| Yaw, a_{66} | 1.20 – 1.75 | 1.40 | 1.53 |

4.0 RESULTS AND DISCUSSIONS

4.1 The Model Ship

This study considers typical inland vessel of length 30.64 meter and 46.8 meters for struck vessel and striking vessel respectively. Figure 3 depicts a typical collision scenario generated using 3D mesh for this particular study.

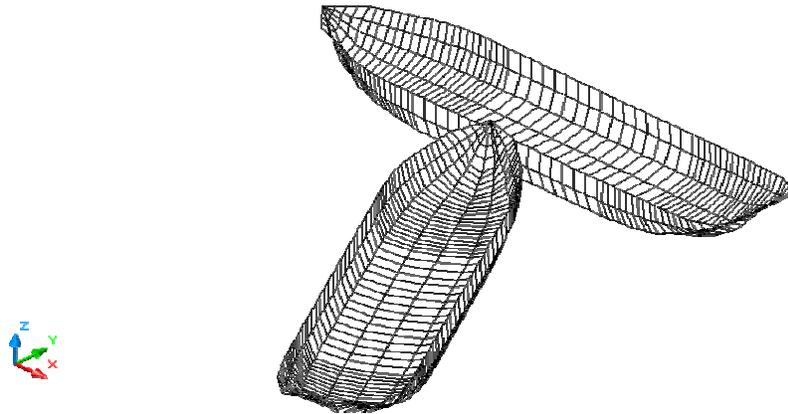


Figure 3: A three dimensional mesh of two ships under collision

The breadth and depth of the struck vessel are 6.7 meter and 3.5 meter respectively. At full load the displacement of the struck ship is 498 tones and the angle of vanishing stability is around 63 degrees in still water condition as shown in Figure 4.

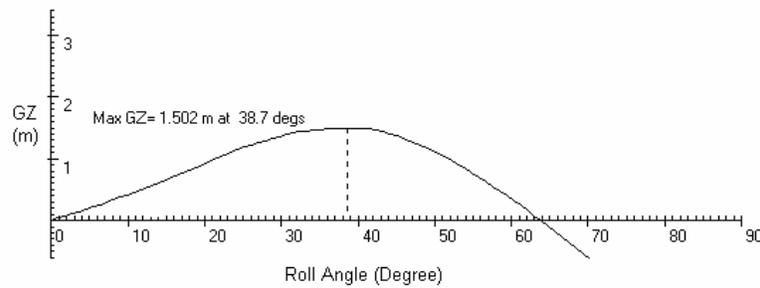


Figure 4: Hydrostatic roll stability of the struck ship

4.2 Results on Maximum Amplitude

Attempt has been taken to investigate the capsizing phenomena considering two variables at a time as shown in the following figures. Figure 5 represents the maximum rolling angle against two different variables namely, the collision time and collision angle. Collision time refers to the time required for compression, restitution and separation of the contact surfaces of the ships. It has been observed from the chart that the relation between collision time and rolling amplitude is exponential and the relation between collision angle and roll amplitude is trigonometric.

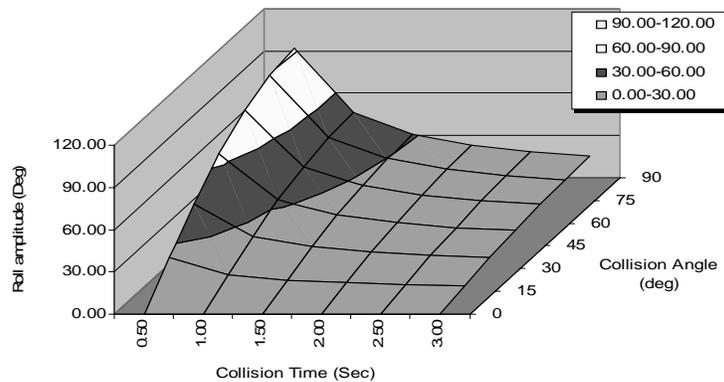


Figure 5: Maximum roll amplitude against collision time and collision angle

The surfaces, as shown by separate colours, represent a particular region for range of rolling amplitude where collision time and collision angle are the variables. In order to find a safe condition, the first and most important parameter is that the rolling amplitude doesn't exceed the angle of vanishing stability. For example in this figure the top two surfaces (roll angle 60 to 90 and roll angle 90 to 120) represent the unsafe condition which exists roughly in the range of 40 deg to 90 deg of collision angle and zero to around 0.8 second of collision time. Therefore, any surface excluding these boundaries represents a safer situation while other variables are kept constant. In this case the ship may survive a collision if it operates or designed with the following parameters: (a) Collision

angle is in between zero to 40 degrees, (b) collision time roughly greater than 0.8 second.

Figure 6 represents a 3D surface chart for a range of coefficient of restitutions (E) and speeds of the striking ship (V_b). It is observed that both the variables influence the rolling amplitude linear proportionally. The range for safe surface is observed to exist in between the following variables: (a) The coefficient of restitution in between 0 to 0.62 and (b) striking speed less than 2.5 meters/second (5 knots).

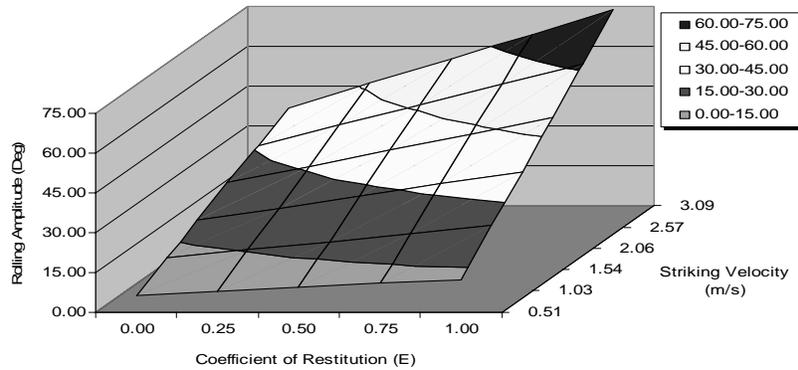


Figure 6: Maximum roll amplitude against coefficient of restitution & striking velocity

Figure 7 shows a surface of collision angle and height of contact point above the centre of flotation. It is observed that the higher the position of the contact point the higher is the amplitude of roll while others variables are kept constant, The amplitude, however, reduces quite significantly with the decrease in the vertical position of the contact point. However, the range of safe surface exists between the following ranges: (a) Collision angle less than 35 degrees and (b) height of collision contact point below 1.75 meters measured above centre of flotation.

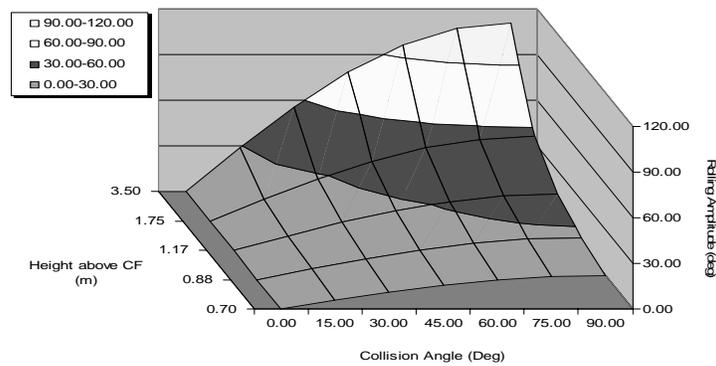


Figure 7: Maximum roll amplitude against collision angle and height of contact point

4.3 Time Domain Simulation

Figure 8 represents the time domain roll simulation of the struck ship and the decay of motion after collision. It is observed that higher striking speeds cause higher the moment for rolling and thus higher rolling amplitude. Although this phenomena is nonetheless a common fact but the key aspect is to observe the amplitudes which are being reduced significantly by alteration of the coefficient of restitution.

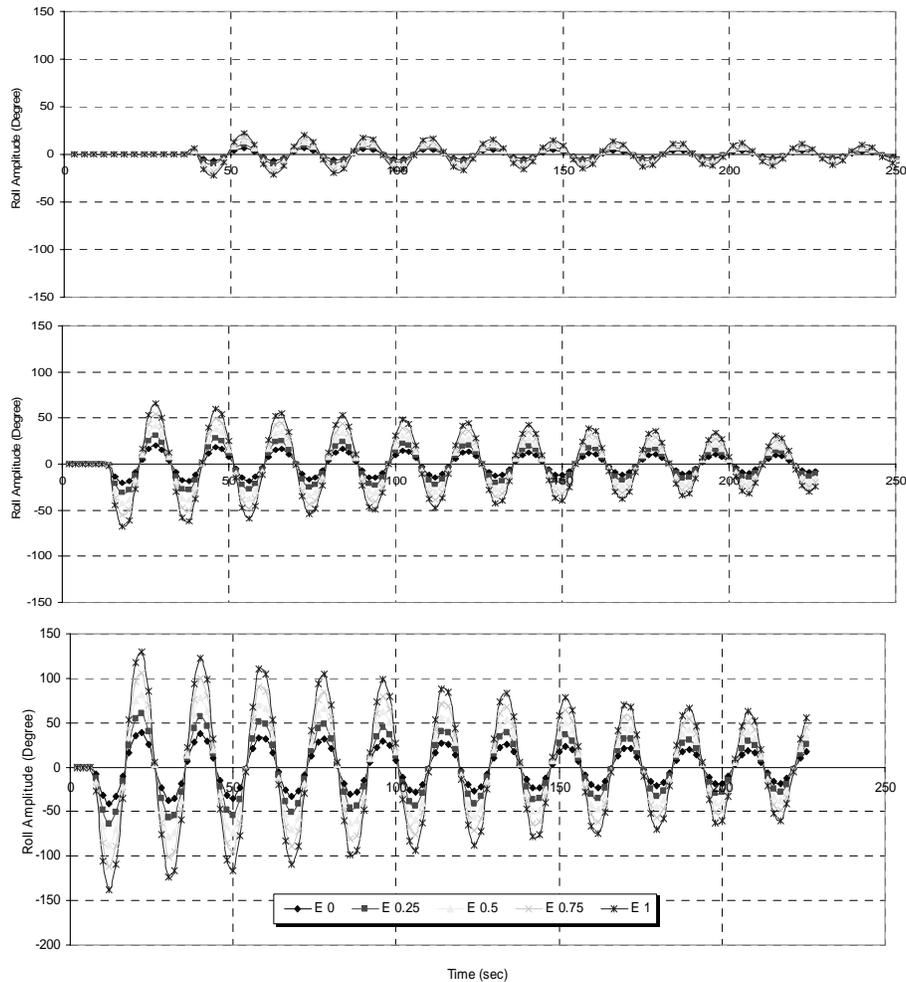


Figure 8: Rolling of struck ship hit 90 deg. at speeds of 1, 3 & 6 knots (top to bottom)

It is observed that up to 83 percent of the rolling amplitude may be reduced if zero restitution materials are being used. This is indeed, a very important aspect of the research findings that as excessive rolling causes ships to capsize and such capsizing could be prevented by applying the lower restitution shock absorbing

materials. This phenomena is simulated the figure where the ship is struck at 6 knots and rolled over the angle of vanishing stability considering the value coefficient of restitution around 1.0. The roll amplitudes, however, are significantly less in their respective cases if the coefficient of restitutions were considered zero or close to zero. Therefore, the facts revealed here could be a mater of life and death and indeed requires recognition to be looked into while construction ship fenders and other similar protective devices.

5.0 CONCLUSIONS

Based on the results, it could be concluded that vessels of this particular type plying in relatively calm waters may use fenders made of materials having coefficient of restitution less than 0.5 and restitution time greater than 1 second to avoid consequences due to collision with a similar vessel. In addition, the collision angle has to be less than 33 degrees, the relative struck speed has to be less than 2.5 meters per second and the vertical collision contact point has to be less than 1.75 meters above the centre of floatation.

The research on studying the capsizing of ships due to collision is still in the initial developing stage. The application of the parameter of coefficient of restitution of the hull material for collision capsizing analysis is considered fundamentally new. So far limited knowledge is available to the researchers about its affect on ship's dynamic behaviour. Further research on this model is therefore recommended, as it seems highly potential in terms of suggesting survivability boundaries for various hazardous operating conditions and thereby, saving invaluable human lives and resources.

NOMENCLATURE

| | |
|-----------|--|
| θ | = Angle between X -axis and I -axis |
| φ | = Angle between X -axis and I -axis |
| b_{ij} | = Damping force coefficient |
| c_{ij} | = Restoring force coefficient |
| E | = Co-efficient of restitution |
| F_1 | = Force in the direction of I -Axis |
| F_2 | = Force in the direction of 2 -Axis |
| F_{A1} | = Force on Ship A in the direction of I -Axis |
| F_{A2} | = Force on Ship A in the direction of 2 -Axis |
| F_{B1} | = Force on Ship B in the direction of I -Axis |
| F_{B2} | = Force on Ship B in the direction of 2 -Axis |
| F_I | = Force in the direction of I -Axis |
| F_J | = Force in the direction of J -Axis |
| F_X | = Force in the direction of X -Axis |
| F_Y | = Force in the direction of Y -Axis |
| $'_i'$ | = Motion in ' i ' direction due to force/moment in ' j ' direction |
| KE_{A1} | = Loss of kinetic energy of Ship A along I -axis |
| KE_{A2} | = Loss of kinetic energy of Ship A along 2 -axis |
| KE_{B1} | = Loss of kinetic energy of Ship B along I -axis |

| | |
|-------------------------|--|
| KE_{B1} | = Loss of kinetic energy of Ship B along 2-axis |
| α | = $-\frac{b_{ij}}{2M_V}$ |
| β | = $\sqrt{\frac{c_{ij}}{M_V} - \left(\frac{b_{ij}}{2M_V}\right)^2}$ |
| M_A | = Mass of Ship A |
| M_B | = Mass of Ship B |
| M_V | = Ships Virtual mass/virtual mass moment of inertia |
| T_{col} | = Collision time |
| V_A | = Forward velocity of Ship A |
| $V_{A1} = V_{B1}$ | = Common velocity in the direction of 1-axis |
| $V_{A1 \text{ after}}$ | = Velocity of ship A in the direction of 1-axis after collision |
| $V_{A1 \text{ before}}$ | = Velocity of ship A in the direction of 1-axis before collision |
| $V_{A2} = V_{B2}$ | = Common velocity in the direction of 2-axis |
| $V_{A2 \text{ after}}$ | = Velocity of ship A in the direction of 2-axis after collision |
| $V_{A2 \text{ before}}$ | = Velocity of ship A in the direction of 2-axis before collision |
| V_B | = Forward velocity of Ship B |
| $V_{B1 \text{ after}}$ | = Velocity of ship B in the direction of 1-axis after collision |
| $V_{B1 \text{ before}}$ | = Velocity of ship B in the direction of 1-axis before collision |
| $V_{B2 \text{ after}}$ | = Velocity of ship B in the direction of 2-axis after collision |
| $V_{B2 \text{ before}}$ | = Velocity of ship B in the direction of 2-axis before collision |
| $FC_i(t)$ | = Force/Moment due to collision in i direction |

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